Return, Risk, and the Security Market Line

Chapter 13

Outline

1. Expected Returns and Variances
2. Portfolios
3. Risk, Diversification, and Portfolio Risk
4. Beta and the Security Market Line

Expected Returns and Variances

Expected Return

The Expected Return is the return a risky asset is expected to yield in the future.

While we estimated average returns and variances using historical data in the previous chapter, we now do so based on projections of the future.

Risk & Return Example

To illustrate this concept, assume we hold two stocks for one year.

- Stock L is expected to be down 20% during a recession and up 70% during a boom
- Stock U is expected to be up 30% during a recession and up 10% during a boom

Let's find the variance and standard deviation for each stock.

Risk & Return Example

Assume there's a 50% chance that either a recession or boom occurs next year.

State of Economy	Probability of State	Rate of Return L	Rate of Return U
Recession	0.5	-20%	30%
Boom	0.5	70%	10%
	1.00		

$$E(R_L) = -0.2(0.5) + 0.7(0.5) = 0.25 = 25\%$$

$$E(R_U) = 0.3(0.5) + 0.1(0.5) = 0.20 = 20\%$$

Risk & Return Example

We can find the variance and standard deviation as well. Note here, instead of dividing by (T-1), we multiply by the probabilities:

State of Economy	Probability of State	Rate of Return L	Rate of Return U
Recession	0.5	-20%	30%
Boom	0.5	70%	10%
	1.00	E(R) = 25%	E(R) = 20%

$$Var(R_L) = 0.5(-0.20-0.25)^2 + 0.5(0.7-0.25)^2 = 0.2025 \text{ and } SD = 45\%$$

$$Var(R_U) = 0.5(0.30-0.20)^2 + 0.5(0.10-0.20)^2 = 0.01 \text{ and } SD = 10\%$$

Risk & Return Example

Summarizing what we've found:

	Stock L	Stock U
E(R)	25%	20%
Var	0.2025	0.0100
SD	45%	10%

To Summarize

The expected return can be found by summing the probabilities multiplied by returns across states. The variance is found by summing the probabilities multiplied by the squared deviations across states.

Now that we can do this for individual assets, we can think about *portfolios*.

Portfolios

Portfolio of Assets

We've calculated the return and risk for single assets in the previous section, but investors rarely hold just one asset.

We can use similar methods for a Portfolio, or a collection of assets, to see how an investor's investments do overall.

Portfolio Example

Find the portfolio expected return, variance, and standard deviation given the following information, assuming you hold each share in equal proportions.

State of Economy	Probability of State	IBM	Microsoft	Apple
Boom	0.4	10%	15%	20%
Bust	0.6	8%	4%	0%
	1.00			

Portfolio Example: E(R)

State of Economy	Probability of State	IBM	Microsoft	Apple
Boom	0.4	10%	15%	20%
Bust	0.6	8%	4%	0%

$$E(R)_{IBM} = 0.4(0.10) + 0.6(0.08) = 8.8\%$$
$$E(R)_{Microsoft} = 0.4(0.15) + 0.6(0.04) = 8.4\%$$
$$E(R)_{Apple} = 0.4(0.2) + 0.6(0.0) = 8.0\%$$

$$E(R)_{Portfolio} = (1/3)(8.8\%) + (1/3)(8.4\%) + (1/3)(8.0\%) = 8.4\%$$

Portfolio Example: Variance & Standard Deviation

State of Economy	Probability of State	IBM	Microsoft	Apple
Boom	0.4	10%	15%	20%
Bust	0.6	8%	4%	0%

$$E(R)_{Boom} = (1/3)(10\%) + (1/3)(15\%) + (1/3)(20\%) = 15\%$$
$$E(R)_{Bust} = (1/3)(8\%) + (1/3)(4\%) + (1/3)(0\%) = 4\%$$

$$VAR_{Portfolio} = 0.4(15\% - 8.4\%)^2 + 0.6(4\% - 8.4\%)^2 = 0.002904$$
$$SD_{Portfolio} = \sqrt{0.002904} = 5.39\%$$

Portfolio Example: Summary

We've found the following:

$$E(R)_{Portfolio} = 8.4\%$$

$$VAR_{Portfolio} = 0.002904$$

$$SD_{Portfolio} = 5.39\%$$

To Summarize

We can find the expected return of a portfolio by considering the assets' weighting and return in each state. To find the variance and standard deviation, we use the expected return and returns in each state of the entire portfolio.

We can now learn about the benefits of *diversification*.

Risk and Diversification

Returns

Returns consist of an expected and unexpected portion. Let's consider an example.

Expected and Unexpected Returns: Generic Co.

What will determine the price of Generic Co.'s shares in one year? The *expected* portion is what investors generally believe based on their own forecasts or the company's forecasts. The *unexpected* portion is due to surprises in the news that has an affect on Generic Co.'s future viability, cash flows, and value.

$$Total\ Return = R = E(R) + U$$

Unexpected Return

The unexpected portion of the return is the true risk of the investment. If we receive *exactly* what we expect, then the asset is risk-free.

Furthermore, there are two components to this "surprise" component of the return: systematic and unsystematic risk.

(1) Systematic Risk

Systematic Risk affects many assets. These are surprises that influence the market overall.

Example: A news report that GDP is unexpectedly falling and unemployment is unexpectedly rising hurts Generic Co. and just about all other companies nation-wide.

(2) Unsystematic Risk

Unsystematic Risk affects, at most, a small number of assets. These surprises are sometimes called *unique* or *firm-specific* risk.

Example: Assume Generic Co. is an oil refinery company and they unexpectedly strike a massive oil reserve on their land. This wouldn't necessarily echo across the country, but would be quite good for Generic Co.

Systematic and Unsystematic Returns

We can now rewrite the surprise component of returns:

$$R = E(R) + U$$
$$R = E(R) + Systematic\ Portion + Unsystematic\ Portion$$
$$R = E(R) + m + \epsilon$$

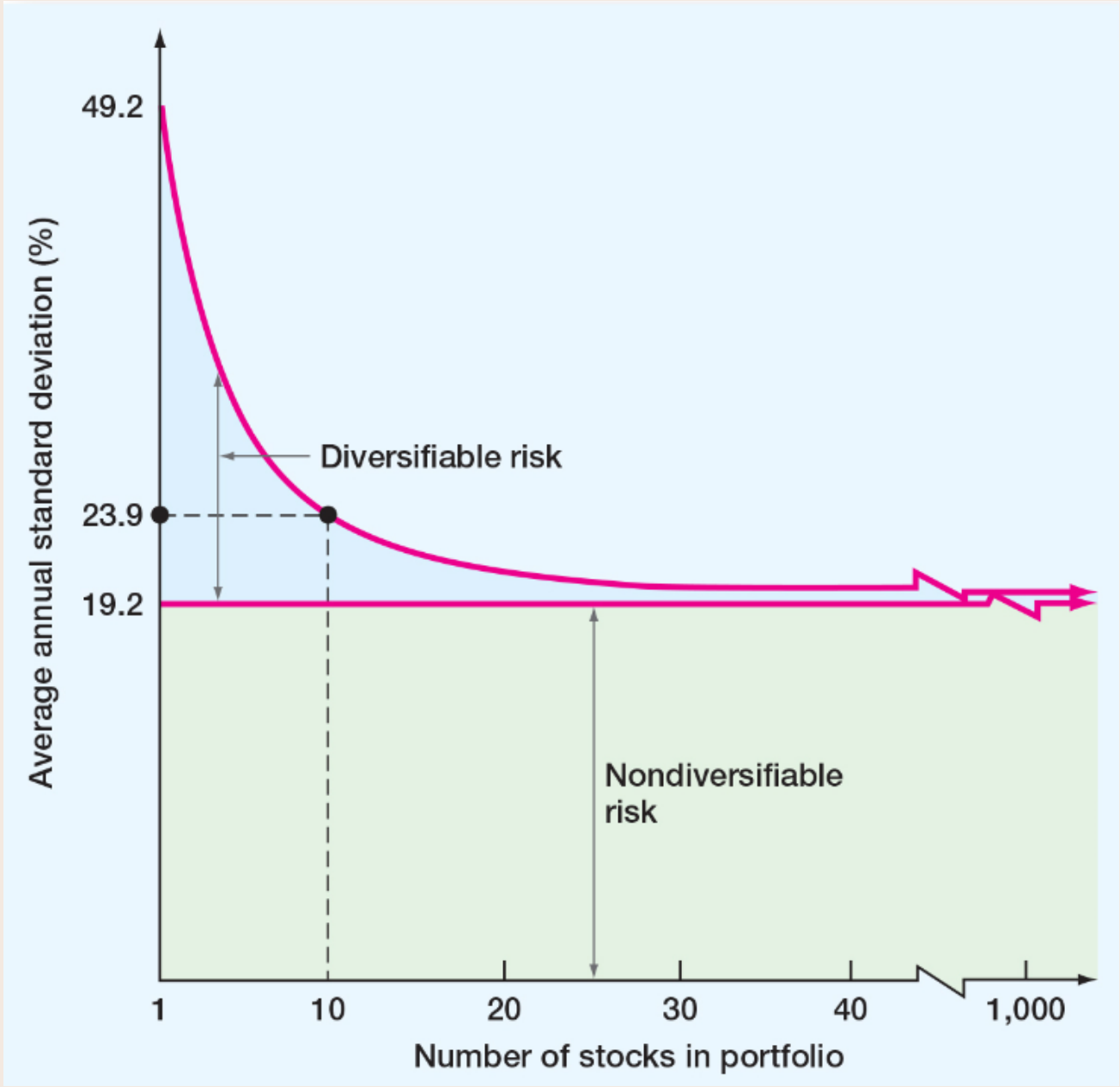
Now that we understand risk, what can we do to reduce it?

Diversification

Diversification involves spreading an investment across a number of assets. Doing so will eliminate some, but not all, risk.

Specifically, diversification helps to eliminate the unsystematic portion of risk.

(1) Number of Stocks in Portfolio	(2) Average Standard Deviation of Annual Portfolio Returns	(3) Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock
1	49.24%	1.00
2	37.36	.76
4	29.69	.60
6	26.64	.54
8	24.98	.51
10	23.93	.49
20	21.68	.44
30	20.87	.42
40	20.46	.42
50	20.20	.41
100	19.69	.40
200	19.42	.39
300	19.34	.39
400	19.29	.39
500	19.27	.39
1,000	19.21	.39


49.2
23.9
19.2
Average annual standard deviation (%)
Diversifiable risk
Nondiversifiable risk
1
10
20
30
40
1,000
Number of stocks in portfolio

Diversification

Holding a portfolio of many assets effectively cancels out any firm specific risk.

Unsystematic risk is essentially eliminated by diversification, so a portfolio with many assets has almost no unsystematic risk.

To Summarize

The return of an asset has a systematic and unsystematic (or firm specific) portion. Diversification practically eliminates firm specific risk.

We now move on to quantifying risk with the understanding that firm specific risk can be eliminated.

Beta

Systematic Risk

The Systematic Risk Principle is the idea that the expected return on a risky asset depends only on that asset's systematic risk.

This is because unsystematic risk can be eliminated by diversifying. The market doesn't reward risks that are borne unnecessarily.

Beta

The Beta Coefficient measures the amount of systematic risk present in a particular asset relative to that in an average risky asset.

The average asset has a beta of 1.

Apple

Netflix

Disney

Coca-Cola

Portfolio Beta Example

What is the expected return and beta of this portfolio?

Security	Amount Invested	Expected Return	Beta
Stock A	$1,000	8%	0.8
Stock B	$2,000	12%	0.95
Stock C	$3,000	15%	1.10
Stock D	$4,000	18%	1.40

Portfolio Beta Example

Security	Amount Invested	Expected Return	Beta
Stock A	$1,000	8%	0.8
Stock B	$2,000	12%	0.95
Stock C	$3,000	15%	1.10
Stock D	$4,000	18%	1.40

We see that we stocks A, B, C, and D are 10%, 20%, 30%, and 40% of the portfolio.

$$E(R)_{Portfolio} = 0.1(8\%) + 0.2(12\%) + 0.3(15\%) + 0.4(18\%) = 14.9\%$$

$$E(\beta)_{Portfolio} = 0.1(0.8) + 0.2(0.95) + 0.3(1.10) + 0.4(1.40) = 1.16$$

This portfolio has greater systematic risk than the average asset.

To Summarize

The beta coefficient measures the amount of systematic risk a security or portfolio possesses.

With this understanding, we can now think about how risk is rewarded in the market place.

The Security Market Line

Beta and the Risk Premium Example

Assume you hold some risky asset, Asset A, with an expected return of 20% and beta of 1.6. You create a portfolio with 75% of your holdings in Asset A, and 25% in some risk free asset (say, T-bills) that returns 8%. Find the expected return and beta of this portfolio.

Beta and the Risk Premium Example

$E(R)_{Portfolio} = 0.25(0.2) + 0.75(0.08) = 11\%$

$\beta_{Portfolio} = 0.25(1.6) + 0.75(0) = 0.40$

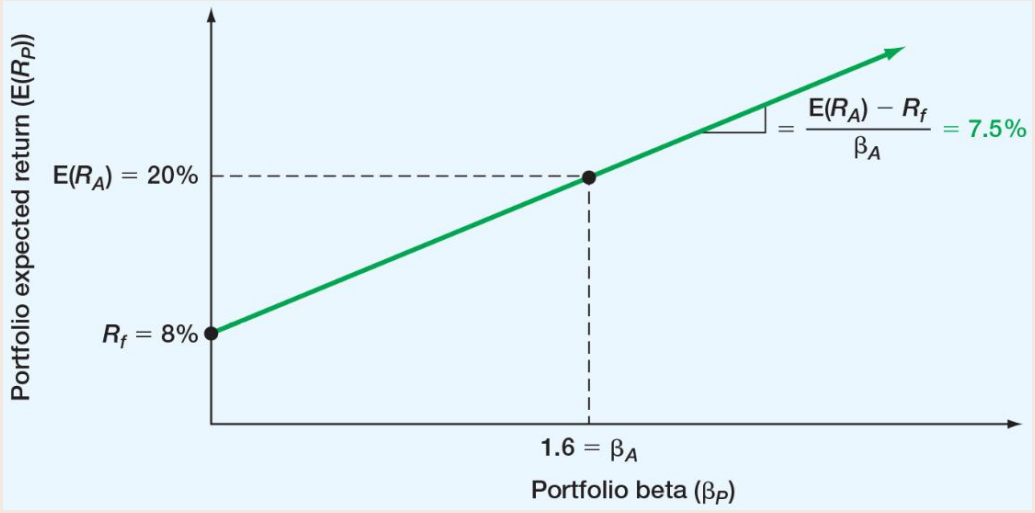
We can compute the expected return and beta for the portfolio at different weightings of Asset A and the Risk Free asset:

50% in A, 50% in Risk Free: $E(R)_{Portfolio} = 14\%$ and $\beta_{Portfolio} = 0.8$

75% in A, 25% in Risk Free: $E(R)_{Portfolio} = 17\%$ and $\beta_{Portfolio} = 1.2$

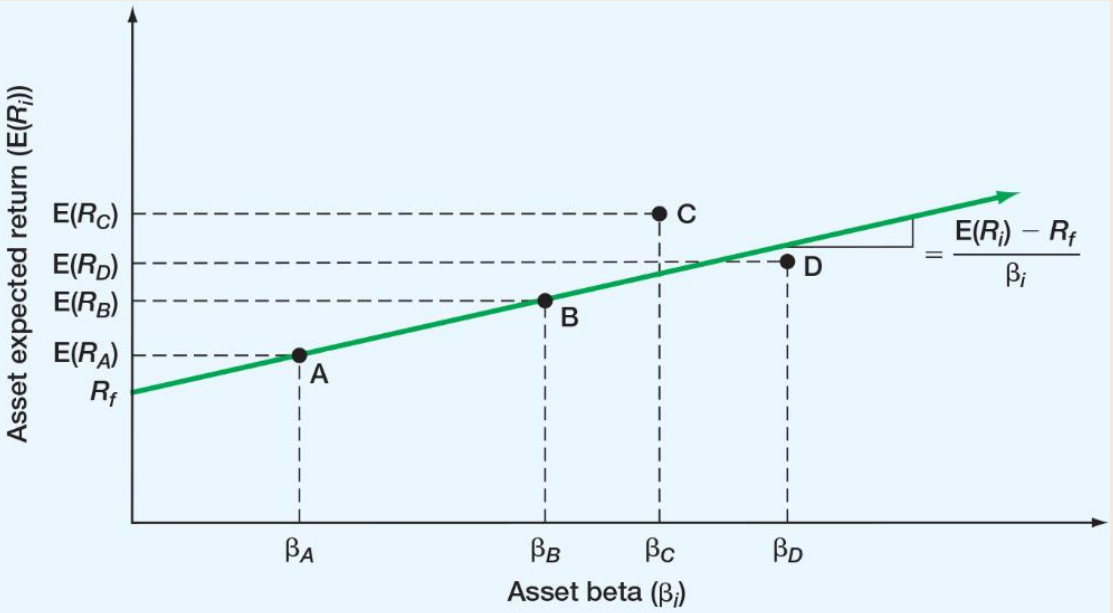
Beta and the Risk Premium Example

Let's plot all these different combinations on a plot with the y-axis the portfolio expected returns and the x-axis the portfolio betas. The slope is the "rise over run" also known as the Reward to Risk Ratio.



Beta and the Risk Premium Example

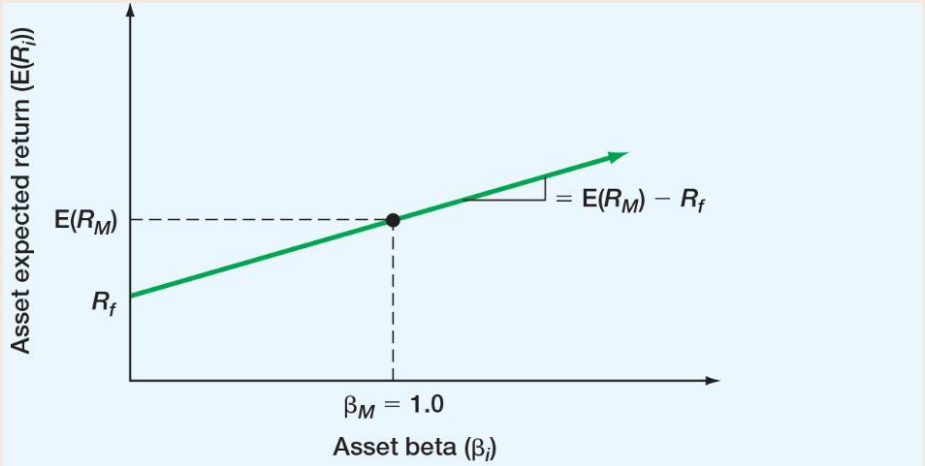
We can include any number of assets on this line. **All reward-to-risk ratios must be the same for all assets**, otherwise investors would "buy up" undervalued assets until they were properly priced.



The fundamental relationship between beta and expected return is that all assets must have the same reward-to-risk ratio, $[E(R_i) - R_f]/\beta_i$. This means that they would all plot on the same straight line. Assets A and B are examples of this behavior. Asset C's expected return is too high; asset D's is too low.

The Security Market Line (SML)

The Security Market Line is the positively sloped straight line displaying this relationship between expected return and beta.



The slope of the security market line is equal to the market risk premium—that is, the reward for bearing an average amount of systematic risk. The equation describing the SML can be written:

$$E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

which is the capital asset pricing model (CAPM).

The Security Market Line (SML)

For the market portfolio, we know beta is equal to 1 because it must have the average systematic risk.

$$SML\ Slope = \frac{E(R_M) - R_f}{\beta_M} = \frac{E(R_M) - R_f}{1} = E(R_M) - R_f$$

The expected market return minus the risk free rate is the Market Risk Premium.

The Capital Asset Pricing Model

We know that all assets must plot on the SML, or their reward to risk ratio must be equal to the slope of the SML. Therefore we have:

$$\frac{E(R_i) - R_f}{\beta_i} = E(R_M) - R_f$$

Which rearranges to:

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f)$$

The Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f)$$

The CAPM shows that the expected return depends on

(1) Pure time value of money through the risk free rate

(2) The reward for bearing systematic risk (the market risk premium)

(3) The amount of systematic risk (β)

The Capital Asset Pricing Model: Example

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f)$$

Assume the risk free rate is 4%, the market risk premium is 8.6%, and a particular stock has a beta of 1.3. What is the expected return on this stock?

$$E(R_i) = 0.04 + 1.3\,(0.086) = 15.18\%$$

To Summarize

The SML shows the relationship between expected return and beta. These concepts of risk and return will be used in the next chapter to determine the value of a company overall.

Takeaways

Takeaways

1. Investors are only rewarded for taking systematic, or non-diversifiable, risk.
2. Beta measures the amount of systematic risk in an asset relative to the average risky asset.
3. The SML displays the relationship between expected return and beta. All risky assets should lie on this line.
4. The CAPM comes from the understanding that the reward to risk ratio for all assets should equal the slope of the SML.
5. Understanding these relationships between risk and return will allow us to set suitable discount rates for valuing a company.

End.