The Time Value of Money

Chapter 5

OUTLINE

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2. Future Value and Compounding
3. Present Value and Discounting
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The Time Value of Money

The Time Value of Money

In general, a dollar today is worth more than a dollar in the future.

A Simple Example

You win a raffle for $1,000. You can either pick up the money today or in one year. A local bank pays 5% interest per year. Does it matter when you collect the prize money?

If you collect today, it is worth $1,000 plus 5% of $1,000 ⟶ $1,050 after one year.

If you collect in one year, it is still worth $1,000 when you pick it up.

Obviously, take the money now!

How Important is This?

This is the *single most important* concept in this course, and a central theme in finance.

Finance applications:

- Stock valuation
- Bond valuation
- Project valuation
- Company valuation

Other applications:

- Buying a house
- Saving for retirement
- Pursuing an advanced degree

To Summarize

A dollar today is worth more than a dollar in the future!

Future Value and Compounding

Future Value

The amount an investment is worth after one or more periods.

Raffle Example Revisited

You win a raffle for $1,000. A local bank pays 5% interest per year. You collect the money today and keep it at the bank for 1 year.

What do think the Future Value (FV) is?

$$
\begin{aligned}
FV &= \$1,000 \;+\; \$1,000 \times (0.05) \\
&= \$1,000 \times (1 + 0.05) \\
&= \$1,000 \times (1.05) \\
&= \$1,050
\end{aligned}
$$

The future value of $1,000 invested for one year at 5% is $1,050.

RAFFLE EXAMPLE REVISITED

What if you collect the money and decide to keep it at the bank for *two* years?

What do think the Future Value (FV) is?

$$
\begin{aligned}
FV_{One\ Year} &= \$1,000 \ + \ \$1,000 \times (0.05) \\
&= \$1,000 \times (1 + 0.05) \\
&= \$1,000 \times (1.05) \\
&= \$1,050
\end{aligned}
$$

$$
\begin{aligned}
FV_{Two\ Years} &= \$1,050 \ + \ \$1,050 \times (0.05) \\
&= \$1,050 \times (1 + 0.05) \\
&= FV_{One\ Year} \times (1 + 0.05) \\
&= \$1,000 \times (1.05) \times (1.05) \\
&= \$1,102.50
\end{aligned}
$$

The future value of $1,000 invested for two years at 5% is $1,102.50.

Compound Interest

We see from the example that we earn *interest on our interest* as well as interest on the original investment, the *principal*.

This is known as *compound interest*.

Compounding Over Many Periods

What if you collect the $1,000 and decide to keep it at the bank for 10 years?

$$FV_{One\ Year} = \$1,000 \times (1.05)$$

$$FV_{Two\ Years} = \$1,000 \times (1.05) \times (1.05)$$

$$FV_{Three\ Years} = \$1,000 \times (1.05) \times (1.05) \times (1.05)$$

$$\ldots$$

$$FV_{Ten\ Years} = \$1,000 \times (1.05)^{10} = \$1,628.89$$

Calculating the Future Value

$$FV = PV(1 + r)^t$$

The future value of a sum is equal to the value of the sum today (the present value or principal, *PV*) times 1 plus the interest rate *r* raised to the number of compounding periods *t*. The expression $(1 + r)^t$ is known as the *future value interest factor*.

Practice Problem: Vacation

You've just earned a bonus of $2,000. You'd like to eventually take a nice vacation, and the total cost of the trip you want is $3,200. The travel agency guarantees that price won't increase for the next 5 years. You found an investment that returns 9% per year. Will you be able to afford the trip with your bonus money before the price goes up?

$$FV = \$2,000(1 \ + \ 0.09)^5 = \$3,077.25$$

You can't afford the vacation!

Practice Problem: Biology

You've engineered a mutant gene that increases in size by 40% every month. It currently has a diameter of 2 nm. How big will the gene be after 8 months?

$$FV = 2\ nm(1\ +\ 0.40)^8 = 29.52\ nm$$

Important: Notice that the rate and the compounding periods are the same unit - months.

Simple Interest

If we do not allow interest to compound, the future value of our investment is much less, particularly for long periods.

Year	Beginning Amount	Simple Interest	Compound Interest	Total Interest Earned	Ending Amount
1	$100.00	$10	$.00	$10.00	$110.00
2	110.00	10	1.00	11.00	121.00
3	121.00	10	2.10	12.10	133.10
4	133.10	10	3.31	13.31	146.41
5	146.41	10	4.64	14.64	161.05
Total		$50	$11.05	$61.05	

To Summarize

We calculate the *future value* of the *principal* by multiplying *1 + the growth rate* raised to the number of compounding periods.

Present Value and Discounting

The Present Value

The current value of future cash flows *discounted* at the appropriate *discount* rate.

Vacation Example Revisited

You know you can invest your vacation money and earn 9% per year. How much money do you need to put into your investment today in order to have \$3,200 in 5 years?

$$FV = PV(1 + r)^t$$

$$\$3,200 = PV(1 + 0.09)^5$$

$$PV = \frac{\$3,200}{(1 + 0.09)^5}$$

$$PV = \$2,079.78$$

Thus, the present value of \$3,200 discounted back 5 years at 9% per year is \$2,079.78. Save this amount to afford your vacation.

Discounting

To *discount* is to calculate the value today of some future amount. The *discount rate* is the rate you used to calculate the present value.

Think of this as the opposite of *compounding*.

Calculating the Present Value

$$PV = \frac{FV}{(1 + r)^t}$$

The present value of a sum is equal to the value of a sum in the future (the lump sum or future value, *FV*) divided by 1 plus the discount rate *r* raised to the number of discounting periods *t*. The expression $1/(1+r)^t$ is known as the *present value interest factor*.

Practice Problem: Funding Retirement Bonuses

The board of directors of the company you work for approaches you with a problem. They estimate 17 employees will be retiring in exactly 8 years, and that each is due a retirement bonus of $1,000 at that time. The board has a safe investment account where they can earn 4% per year. How much do they need to put into this account to be able to pay those retirement bonuses?

$$PV = \frac{FV}{(1 + r)^t}$$

$$PV = \frac{17 \times \$1,000}{(1 + 0.04)^8} = \$12,421.73$$

To Summarize

The present value is the *discounted* value of some future sum, which we find by dividing that value by *1 + the discount rate* raised to the number of discounting periods.

Applications

Using a Financial Calculator: The TI BA II Plus

1. Display the maximum number of decimal places:

   2ND ⟶ FORMAT ⟶ use arrows to navigate to DEC ⟶ 9 ⟶ ENTER

2. For now, make sure there is no "BGN" above your zero on your screen:

   2ND ⟶ BGN ⟶ 2ND ⟶ SET

3. Before *each* TVM problem, clear your work:

   2ND ⟶ CLR TVM

4. Always remember to put a negative sign in front of cash outflows.

Practice Problem: Lawsuit

You've been sued! You have to pay the $10,000 settlement in 3 years. How much do you have to put away today if you can earn 3% at the bank per year?

This is a PV problem.

2ND $\longrightarrow$ CLR TVM
N = 3 $\longrightarrow$ the number of years
I/Y = 3 $\longrightarrow$ the interest rate
FV = -10000 $\longrightarrow$ the future value, a cash outflow
CPT PV = 9151.41

You should put away at least $9,151.41.

PRACTICE PROBLEM: BUYING A MOTORCYCLE

The Scout FTR 1200 is being released in 4 years and will cost $13,000. You put $11,000 in an investment that yields 8% annually. Will you be able to afford it when it's released?

This is a FV problem.

2ND ⟶ CLR TVM
N = 4 ⟶ the number of years
I/Y = 8 ⟶ the interest rate
PV = -11000 ⟶ the future value, a cash outflow
CPT FV = 14965.38

You'll have $14,965.38, which is enough to pay for the $11,000 bike.

Practice Problem: Saving for College

When you have your first child, you will put away \$30,000 to save for their college which you estimate will cost \$100,000 in 18 years. What annual rate of interest do you need to earn in order to be able to afford tuition at that time?

Here, find the interest rate.

2ND ⟶ CLR TVM
N = 18 ⟶ the number of years
PV= -30000 ⟶ the present value, a cash *outflow* (giving money to the bank)
FV = 100000 ⟶ the future value, a cash *inflow* (getting money from the bank)
CPT I/Y = 6.92

You'll need to save at an annual rate of 6.92%.

Practice Problem: Tricky Timelines

How much do you need to invest in 3 years if you plan on having $30,000 in 10 years? You'll invest at 12% per year.

This is a PV problem.

2ND $\longrightarrow$ CLR TVM
N $= 10 - 3 = 7$
I/Y $= 12$
FV $= 30000 \longrightarrow$ a cash inflow
CPT PV $=$ -13,570.48 $\longrightarrow$ a cash *outflow* that you are depositing in an investment account.

You'll need to put away $13,570.48 in 3 years to have $30,000 in 10 years.

Practice Problem: A Signing Bonus

In 2 years, you will graduate and your employer will give you a signing bonus of 10% of your $48,000 salary. At that time, you plan on investing it at 4% per year until you have $12,000, enough for you to take a 3 year backpacking trip. In how many years from now will you be able to afford this trip?

Here, find the number of years plus 2, the number of years until you get your signing bonus.

2ND → CLR TVM
PV = -48000 * 10% = -4800 → a cash *outflow*
I/Y= 4
FV = 12000 → a cash *inflow*
CPT N = 23.36

You'll need to leave this in your account for 23.36 years, but you won't get your bonus for 2 more years. You can afford the trip in 25.36 years.

Takeaways

Takeaways

1. A dollar today is worth more than a dollar in the future.
2. The future value is a function of the present value, the interest rate, and the number of compounding periods.
3. The present value is a function of the future value, the discount rate, and the number of discounting periods.
4. Applications extend beyond just finance.

End.