Discounted Cash Flow Valuation

Chapter 6

Outline

1. FV of Multiple Cash Flows
2. PV of Multiple Cash Flows
3. Annuities and Perpetuities
4. Interest Rates
5. Types of Loans

FV of Multiple Cash Flows

FV and PV Review

In the previous section, we've calculated the FV and PV for *one* lump sum using the following formulas:

$$FV = PV(1 + r)^t$$

$$PV = \frac{FV}{(1 + r)^t}$$

But many investments have multiple cash flows:

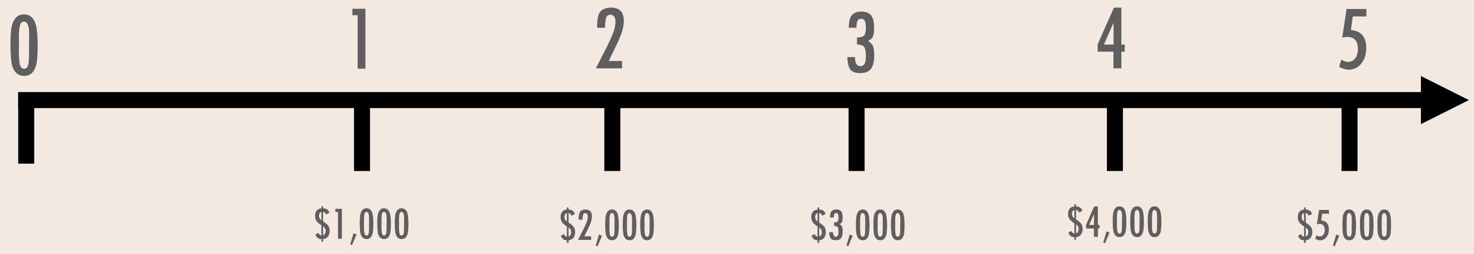
- Example: Costco builds a new store that results in sales each year for many years.

FV of Multiple Cash Flows

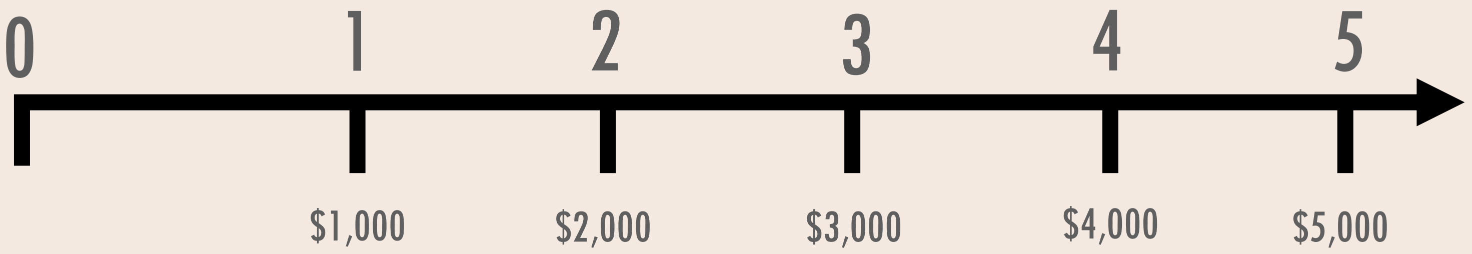
To find the FV of multiple sums over time, find the sum of the individual future values.

FV of Multiple Cash Flows: First Payment Later

What is the value in 5 years of a series of 5 payments to a bank account earning 10% annually, assuming the payments start at $1,000 and increase by $1,000 each year? You make your first payment at the end of this year.



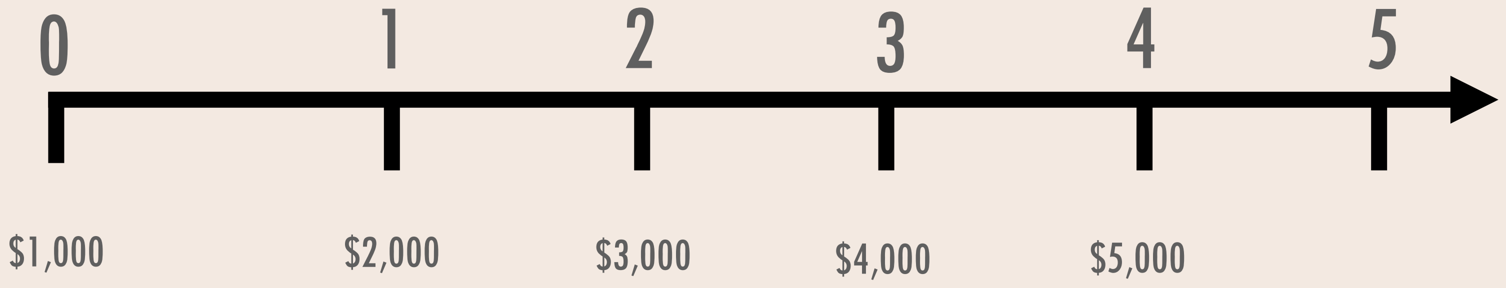
FV of Multiple Cash Flows: First Payment Later



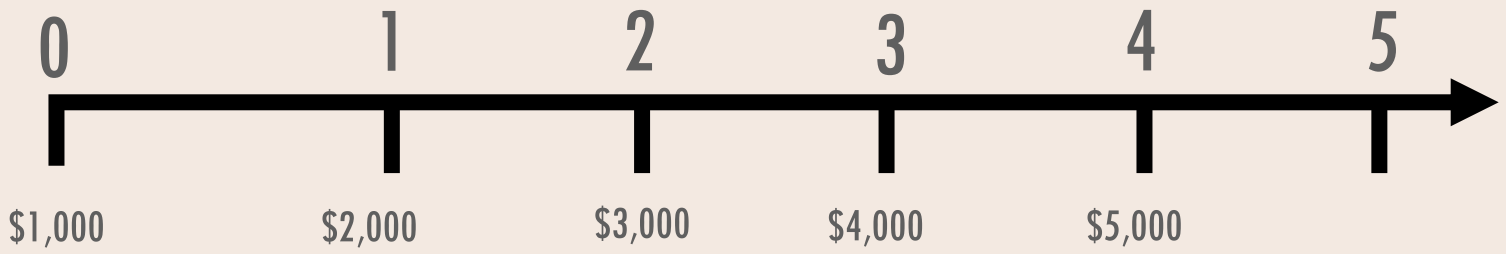
$\$1,000 \times 1.1^4$	=	\$1,464.10
$\$2,000 \times 1.1^3$	=	\$2,662.00
$\$3,000 \times 1.1^2$	=	\$3,630.00
$\$4,000 \times 1.1^1$	=	\$4,400.00
\$5,000	=	\$5,000.00
		\$17,156.10

FV of Multiple Cash Flows: First Payment Today

What is the value in 5 years of a series of 5 payments to a bank account earning 10% annually assuming the payments start at $1,000 and increase by $1,000 each year? You make your first payment at the beginning of this year.



FV of Multiple Cash Flows: First Payment Today



$1,000 X 1.1^5 = $1,610.51

$2,000 X 1.1^4 = $2,928.20

$3,000 X 1.1^3 = $3993.00

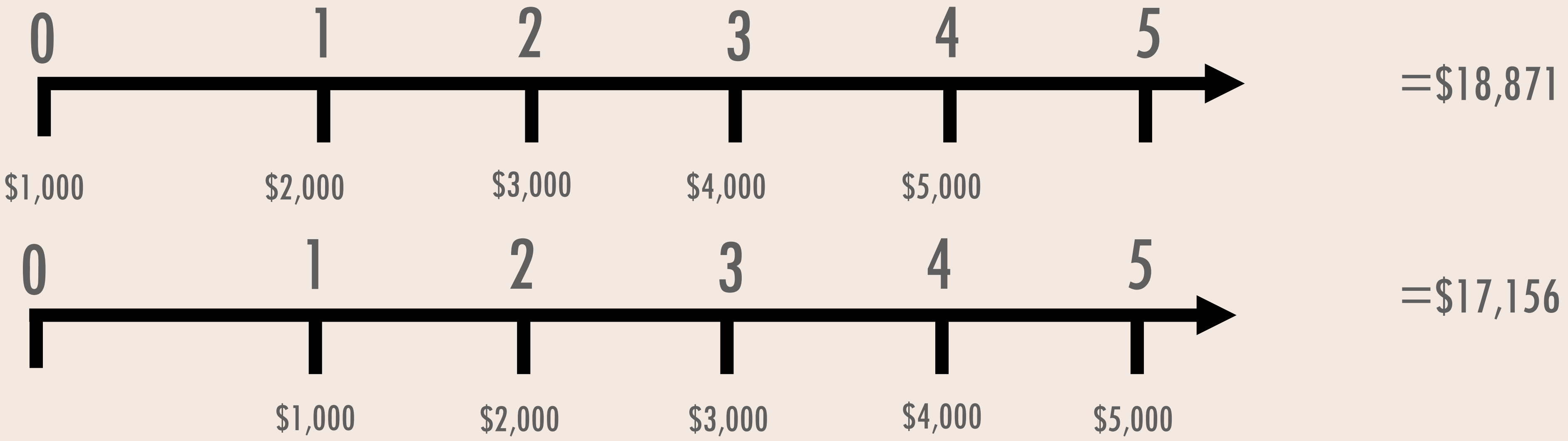
$4,000 X 1.1^2 = $4,840.00

$5,000 X 1.1^1 = <u>$5,500.00</u>

<u>$18,871.71</u>

Timing of Payment Matters!

In the two previous examples, the only thing that changed was the timing, and we had quite different results.



To Summarize

The future value of a series of amounts is the sum of each amount's future value.

PV of Multiple Cash Flows

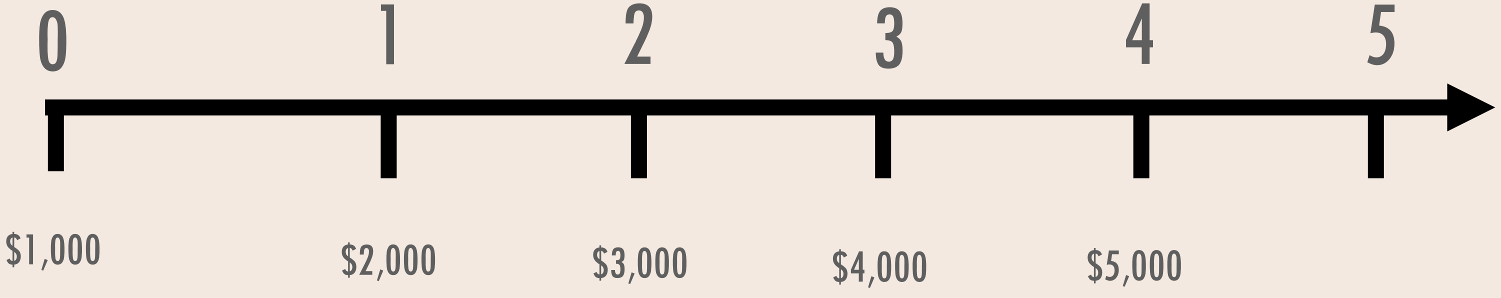
PV of Multiple Cash Flows

To find the PV of multiple sums over time, find the sum of the individual present values.

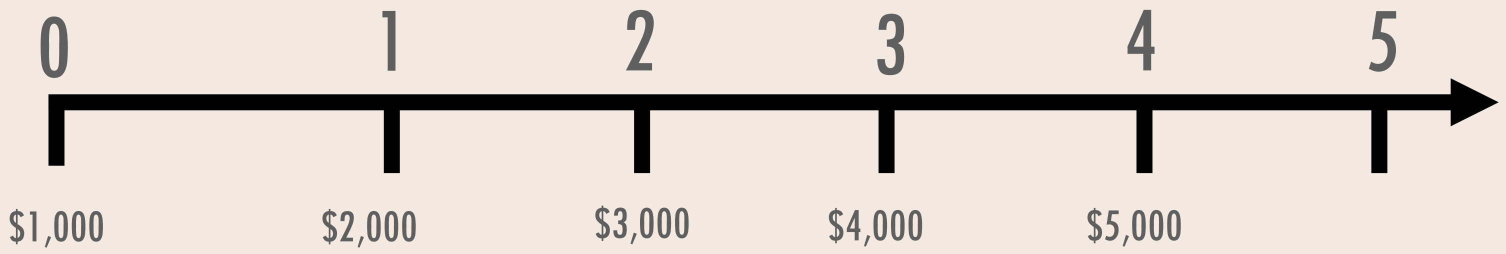
$$PV = \frac{FV}{(1 \ + \ r)^t}$$

PV of Multiple Cash Flows: First Payment Today

What is the value today of a series of 5 payments made to a bank account earning 10% annually assuming the payments start at $1,000 and increase by $1,000 each year? The first payment is at the beginning of this year.



PV of Multiple Cash Flows: First Payment Today



$\$1{,}000 / 1.1^{0} = \quad \$1{,}000.00$

$\$2{,}000 / 1.1^{1} = \quad \$1{,}818.18$

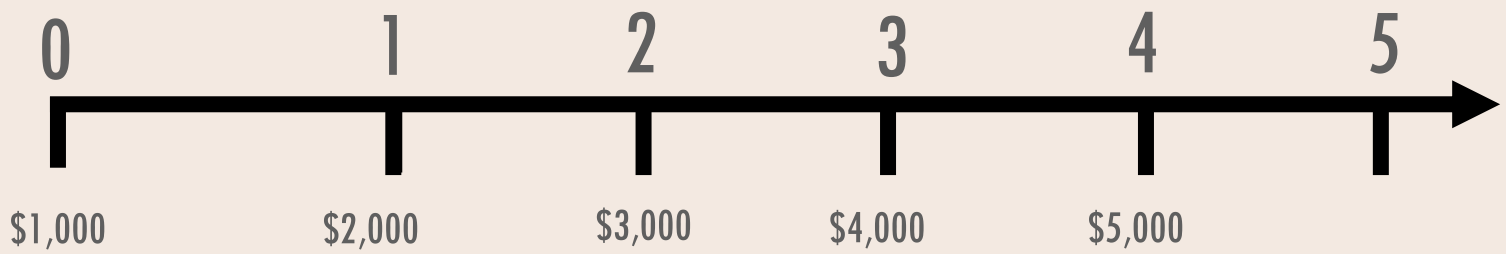
$\$3{,}000 / 1.1^{2} = \quad \$2{,}479.34$

$\$4{,}000 / 1.1^{3} = \quad \$3{,}005.26$

$\$5{,}000 / 1.1^{4} = \quad \underline{\$3{,}415.07}$

$\underline{\$11{,}717.85}$

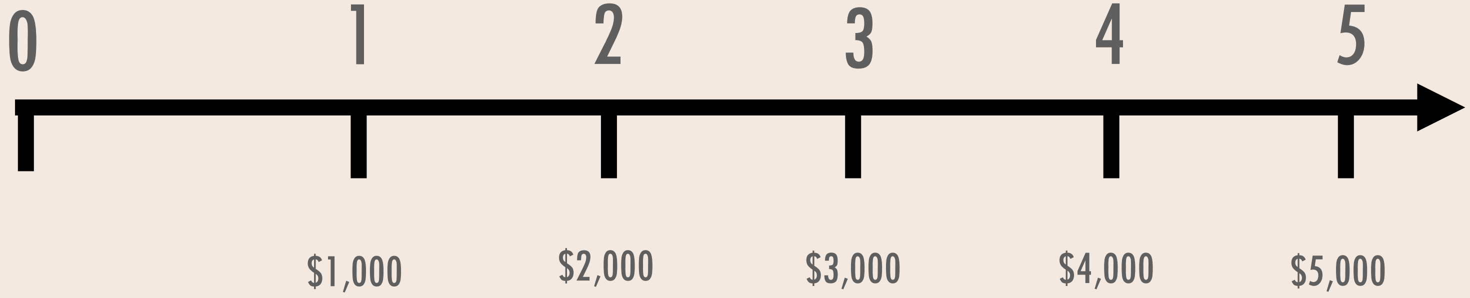
PV of Multiple Cash Flows: Calculator



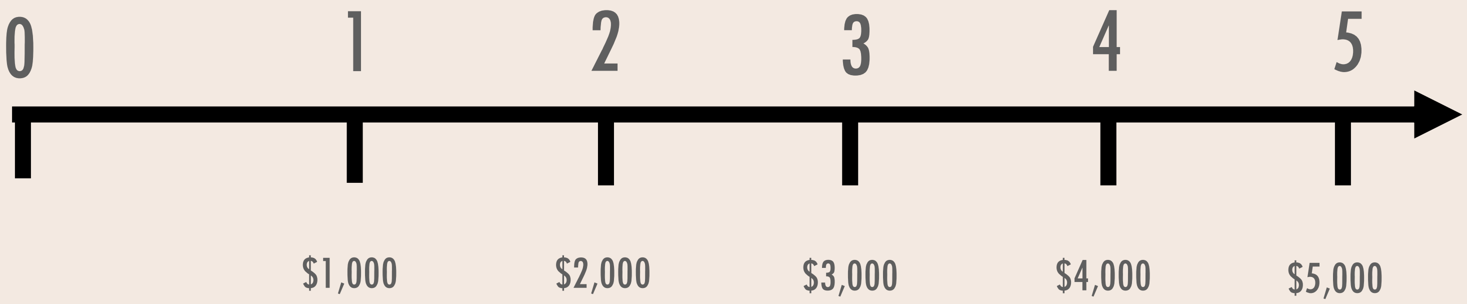
CF
2ND CLR WORK
2ND CLR TVM
CFo = 1000, C01 = 2000, F01 = 1, C02 = 3000, F02 = 1, …, C04 = 5000, F04 = 1
NPV
I = 10
CPT NPV = **11,717.85**

PV of Multiple Cash Flows: First Payment Later

What is the value today of a series of 5 payments made to a bank account earning 10% annually assuming the payments start at $1,000 and increase by $1,000 each year? You make your first payment at the beginning of next year.



PV of Multiple Cash Flows: First Payment Today



$\$1,000 / 1.1^{1} =$ $909.09

$\$2,000 / 1.1^{2} =$ $1,652.89

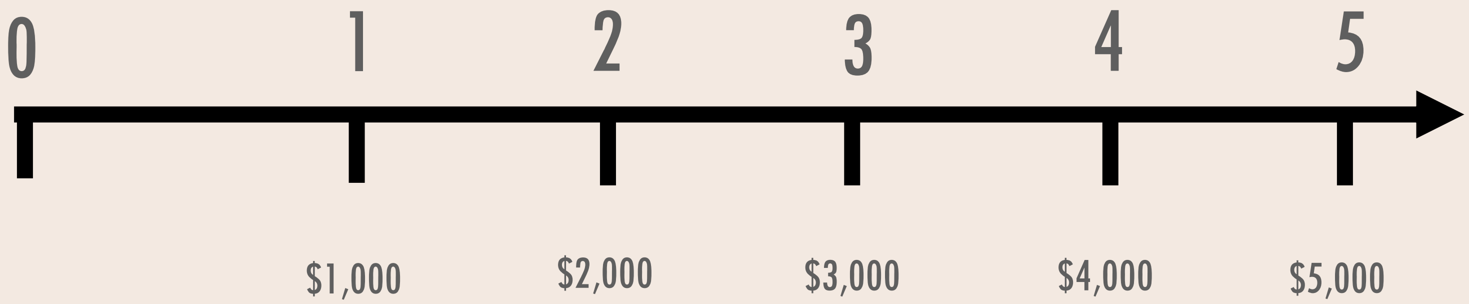
$\$3,000 / 1.1^{3} =$ $2,253.94

$\$4,000 / 1.1^{4} =$ $2,732.05

$\$5,000 / 1.1^{5} =$ $3,104.61

$10,652.58

PV of Multiple Cash Flows: Calculator



CF
2ND CLR WORK
2ND CLR TVM
CFo = 0, C01 = 1000, F01 = 1, C02 = 2000, F02 = 1, …, C05 = 5000, F05 = 1
NPV
I = 10
CPT NPV = **10,652.58**

To Summarize

We find the present value of multiple future cash flows by discounting each of these cash flows to the present and summing them.

Annuities and Perpetuities

Steady Stream of Cash Flows

In the previous examples, the future cash flows could vary in size. We now look at streams of cash flow where the cash flows for each period are the same, or grow by the same amount, each period.

Annuities

A level stream of cash flows over a period of time that either doesn't change or grows at the same rate each period.

Ordinary Annuity: cash flows at the end of each period

Annuity Due: cash flows at the beginning of each period

ORDINARY ANNUITY EXAMPLE

There is an investment opportunity that promises to pay \$500 at the end of each of the next three years. If you want to earn 10%, what's the most you should pay for this investment?

Using the same method we've just learned:

$PV = (\$500/1.1^1)+(\$500/1.1^2)+(\$500/1.1^3) = \$1,243.43$

Ordinary Annuity Example

Because the payments are the same each year, we can use a formula:

$$Ordinary\ Annuity\ PV = C \times \frac{1 - [1/(1+r)^t]}{r}$$

Where C is the payment amount.

ORDINARY ANNUITY EXAMPLE

Plugging in the values:

$$Ordinary\ Annuity\ PV = C \times \frac{1 - [1/(1+r)^t]}{r}$$

$$= 500 \times \frac{1 - [1/(1+0.10)^3]}{0.10}$$

$$= \$1,243.43$$

Ordinary Annuity Example

We can also calculate the PV of these cash flows in our calculator using the PMT key:

N=3

I/Y=10

PMT =500

CPT PV= -$1,243.43

Ordinary Annuity Example 2

You want to purchase the new 256 GB Apple iPhone X that costs $1,149. Assume that you put this on your Chase Freedom credit card, and you can only make the minimum monthly payment of $20. This credit card charges 1.5% a month. How long until you pay off this phone?

PV=1149

I/Y=1.5

PMT=-20

CPT N = 133 months $\approx$ 11 years!

Ordinary Annuity Example 3

Instead of buying that new iPhone, let's invest the $20 a month in the stock market. You expect to earn about 1% a month. How much will you have after 11 years?

We can use the *FV of an Annuity* formula:

$$Ordinary\ Annuity\ FV = C \times \frac{(1+r)^t - 1}{r}$$

$$= 20 \times \frac{(1+0.01)^{11*12} - 1}{0.01} = 5,438$$

Verify in your calculator.

Annuities Due

Here, the first cash flow happens immediately and not at the end of the period.

There's a simple correction:

(1) Calculate the PV or FV as an ordinary annuity

(2) Multiply that PV or FV by (1+r)

In a calculator, switch the mode to BGN.

Annuities Due Example

You want to save up for backstage passes for Beyoncé's upcoming concert. You'll deposit $50 a month starting today. The concert is in 12 months, and you can save at 2% per month. Will you be able to afford the $680 passes?

N=12

PMT=-50

I/Y=2%

CPT FV = 670.60

Then, 670.60 x (1+0.02) = $684.02

Perpetuities

An annuity in which the cash flows continue forever.

$$PV\ of\ a\ Perpetuity = \frac{C}{r}$$

Perpetuities Example

You are interested in buying preferred stock in a company that pays a $3 dividend every quarter. Other companies similar to this one return about 2% every quarter. How much should you pay for the preferred stock?

$$PV\ of\ the\ Perpetuity = \frac{3}{0.02} = \$150$$

PV of Growing Annuities and Perpetuities

You win a lottery that pays you $200,000 at the end of this year. The amount paid each year increases by 5%, so in the 2nd year, you are paid $210,000. In the 3rd year you are paid $220,500, and so on for 20 years. What is the present value if we discount by 11%?

Use the formula:

$$Growing\ annuity\ present\ value = C \times \frac{1 - (\frac{1+g}{1+r})^t}{r - g}$$

PV of Growing Annuities and Perpetuities

$$Growing\ annuity\ present\ value = C \times \frac{1-(\frac{1+g}{1+r})^t}{r-g}$$

$$= 200000 \times \frac{1-(\frac{1+0.05}{1+0.11})^{20}}{0.11-0.05}$$

$$=2{,}236{,}337.06$$

Growing Perpetuities

$$Growing\ perpetuity\ present\ value = \frac{C}{r - g}$$

Growing Perpetuities Example

You have a structured settlement, but you need cash now. You call J.G. Wentworth, 877-CASH-NOW. You explain that you will receive $1,000 in the first year, which grows by 2% per year forever. J. G. Wentworth discounts these cash flows at 4%. How much cash will they give you now?

$$Growing\ perpetuity\ present\ value = \frac{C}{r-g}$$

$$= \frac{\$1,000}{0.04-0.02} = \$50,000$$

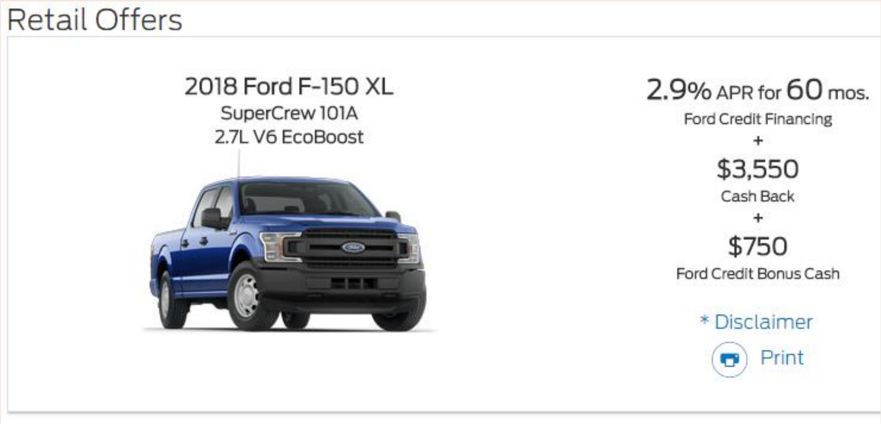
To Summarize

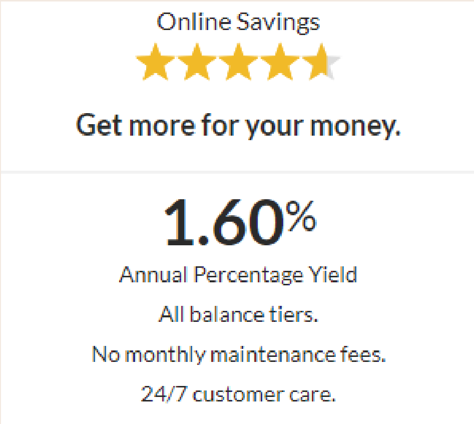
Annuities and perpetuities involve steady cash flows over a period of time or cash flows that grow at a constant rate over time.

Interest Rates

Interest Rates and Compounding

The *quoted* interest rate we see in car and mortgage commercials or on bank websites is generally expressed as an annual rate.





But how often the interest is *compounded* matters substantially.

Interest Rates and Compounding

Example: You see your local bank is quoting a 10% return on an account that *compounds semiannually*. You put $100 in this account. How much interest will you have earned (in percent) by the end of the year?

Because we have 2 six-month periods, the interest rate is 10/2 = 5% for each period

For the first period: $100 x 1.05 = $105
For the second period: $105 x 1.05 = $110.25

The future value is $110.25 which grew from $100. This is a 10.25% increase, not a 10% increase!

APR, EAR, AND APY

The *annual percentage rate* (APR) is the quoted annual rate, or the interest rate charged per period times the number of periods over which it compounds.

The *effective annual rate* (EAR), or the *annual percentage yield* (APY) is the rate you actually pay or earn based on the compounding.

In the previous example, $APR = 10\%$ and $EAR = APY = 10.25\%$.

APR, EAR, and APY Example

You bank offers to pay you 12% compounded quarterly. What is the APR? What is the EAR? What is the APY?

APR = 12%

For the first period:	\$100 x 1.03 = 103
For the second period:	\$103 x 1.03 = 106.09
For the third period:	\$106.09 x 1.03 = 109.2727
For the fourth period:	\$109.2727 x 1.03 = 112.55088

Thus, the EAR = APY = 12.55%

APR, EAR, and APY Example (Calculator)

You bank offers to pay you 12% compounded quarterly. What is the APR? What is the EAR? What is the APY?

APR = 12%

In your calculator:
N = 4, I/Y = 12/4 = 3, PV = -100, CPT FV = 112.55088

Thus, the EAR = APY = 12.55%

APR, EAR, and APY Example 2

You bank offers to pay you 7% compounded daily on a loan that requires a minimum deposit of $6,000. You put in exactly the minimum deposit. What is the APR? What is the EAR? What is the APY? What will be in the account after 2 years?

APR and EAR/APY

APR = 7%

N = 365, I/Y = 7/365 = 0.019178082, PV = -100, CPT FV = 107.2501

EAR = APY = 7.25%

Future Value

N = 365 x 2 = 730, I/Y = 7/365 = 0.019178082, PV = -6000, CPT FV = **$6,901.55**

APR and EAR Conversion

To directly convert a quoted APR to an EAR or APY:

$$EAR = [1 + \frac{APR}{m}]^m - 1$$

Where *m* is the number of compounding periods.
For *continuous* compounding:

$$EAR = e^{APR} - 1$$

Conversion Examples

Convert a quoted rate of 8% compounded weekly to an EAR.

$$EAR = [1 + \frac{APR}{m}]^m - 1$$

$$EAR = [1 + \frac{0.08}{52}]^{52} - 1 = 8.32\%$$

Convert a continuously compounded quoted rate of 14% to an EAR.

$$EAR = e^{APR} - 1$$

$$EAR = e^{0.14} - 1 = 15.03\%$$

Real World Example

You want to purchase a new Porsche for $74,500 and the finance office at the dealership quoted you an APR of 5.6% for a 48 month loan to buy the car. What will your monthly payments be? What is the EAR?

Monthly Payments

N = 48, I/Y = 5.6/12= 0.466667, PV = 74500, CPT PMT = -**1,736.00**

EAR

$EAR = [1 + (.056 / 12)]^{12} - 1 = 5.75\%$

To Summarize

Interest rates are quoted in annual terms, often as an APR. Understanding the number of compounding periods is important for knowing the actual rate we save or borrow at, the EAR or APY.

Types of Loans

Pure Discount Loans

The simplest type of loan- you receive money today and pay back a lump sum in the appropriate number of periods.

Suppose you will receive a bonus of $25,000 in five years, but you want money now. The bank charges 12% on loans. How much will they lend you today if you will pay back in five years the full $25,000?

$$PV = \$25{,}000 / (1.12^5) = \mathbf{\$14{,}186}$$

Interest-Only Loans

Here, the borrower pays interest each period then the entire loan amount or principal in the future.

We will cover this when we get to *bond valuation* in the next chapter.

Amortized Loans

In an amortized loan, a portion of each payment includes interest and a principal reduction.

Mortgages, student loans, credit cards, and car notes are generally amortized loans.

Mortgage Example

You want to obtain a $400,000 loan to purchase a new home. As with most mortgages, the term is 30 years but payments (and compounding) is monthly. The interest rate the bank quotes you is 4.3% per year (the APR). What are your payments?

N = 30 x 12 = 360, I/Y = 4.3/12= 0.35833333, PV = 400000, CPT PMT = -1979.49

You will pay **$1,979.49** a month.

Amortization schedule creator.

Takeaways

Takeaways

1. We obtain the PV or FV of multiple cash flows by summing the PV or FV of each cash flow overtime.
2. The timing of the first payment in a series of cash flows matters.
3. Annuities have constant or constant growth payments overtime.
4. Perpetuities have a steady stream of payments forever.
5. The quoted return and the effective return are different due to compounding.

END.