UNIVERSITY OF RICHMOND
Robins School of Business™

§10. BOND VALUATION

FIN 360: PRINCIPLES OF FINANCIAL MANAGEMENT

TABLE OF CONTENTS

DETERMINING BOND YIELDS

We'll revisit our prior example of the Microsoft corporate bond that they issued for general corporate purposes and acquisitions.

PRACTICE: Microsoft issues a 10-year bond with a 3.3% coupon (or 3.3% ÷ 2 = 1.65% every 6 months) and a $1,000 par value.

a.) Assume you purchase the bond for exactly $1,000 from Microsoft. What is the **yield to maturity (YTM)** – the bond's annual rate of return if you hold it for 10 years?
b.) Instead, assume you purchase the bond on the **secondary market** from another bond investor for $900 immediately after the bond is issued. What is the **yield to maturity (YTM)** in this case?
c.) What if you pay $1,100?

Figure 1: Bond Cash Flows

Microsoft

Year	0	0.5	1	1.5	2	…	8	8.5	9	9.5	10
Coupon		$16.50	$16.50	$16.50	$16.50		$16.50	$16.50	$16.50	$16.50	$16.50
Face Value											$1,000
Cash Flow	*($1,000)*	*$16.50*	*$16.50*	*$16.50*	*$16.50*		*$16.50*	*$16.50*	*$16.50*	*$16.50*	*$1,016.50*

SOLUTION: We determine a bond's value as the

(1) Present Value of the Coupons
(2) Present Value of the Principal

We can use the present value of a perpetuity formula modified to include the present value of the bond's par (or face value):

$$Bond\ Value = \left[C \times \frac{1 - \frac{1}{(1+r)^t}}{r} \right] + \frac{FV}{(1+r)^t}$$

Therefore, we solve for r to find the YTM, making sure to annualize the r we compute.

Part (a), paying $1,000 for the bond

For part (a) where the bond price = $1,000, the coupons are $16.50 (1.65% of the par every 6 months), and the number of periods is 20 (2 semiannual periods × 10 years):

$$\$1{,}000 = \left[16.50 \times \frac{1 - \frac{1}{(1+r)^{20}}}{r}\right] + \frac{1{,}000}{(1+r)^{20}}$$

Solving for r gives

$$r = 0.0165 = 1.65\%$$

The yield to maturity is always expressed as an annual rate, but r was computed based on semiannual payments. Thus, we multiply by 2 to express as an annual rate:

$$YTM = 0.0165 \times 2 = 3.3\%$$

Part (b), paying $900 for the bond

The coupon payments and the par value remain the same.

$$\$900 = \left[16.50 \times \frac{1 - \frac{1}{(1+r)^{20}}}{r}\right] + \frac{1{,}000}{(1+r)^{20}}$$

$$YTM = r \times 2 = 0.02278 \times 2 = 4.556\%$$

Part (c), paying $1,100 for the bond

$$\$1{,}100 = \left[16.50 \times \frac{1 - \frac{1}{(1+r)^{20}}}{r}\right] + \frac{1{,}000}{(1+r)^{20}}$$

$$YTM = r \times 2 = 0.01091 \times 2 = 2.182\%$$

What happens to the yield to maturity the more you pay for the bond? What can we say about the relationship between prices and yields?

We can use our calculators to find the YTM in each of these cases, though we are careful to convert the final I/Y to a YTM by multiplying by 2.

Part (a), paying $1,000 for the bond

N	I/Y	PV	PMT	FV

Part (b), paying $900 for the bond

N	I/Y	PV	PMT	FV

Part (C), paying $1,100 for the bond

N	I/Y	PV	PMT	FV

INTERPRETATION: We can summarize the YTM in these three cases in the following way:

Table 1: Prices and Yields

Bond Price	YTM	Explanation
\$900	4.556%	Purchased **at a discount**: YTM > Coupon Rate.
\$1,000	3.3%	Purchased **at par**: YTM = Coupon Rate.
\$1,100	2.182%	Purchased **at a premium**: YTM < Coupon Rate.

As we've seen before, *yields and prices are inversely related*. Additionally, we can determine if a bond is a discount, at par, or premium bond based on the relationship between YTM and the coupon rate.

Remember that coupon rates are quoted as *annual rates* and need to be converted to *semiannual payments* to input into the calculator. Then, the I/Y the calculator produces needs to be multiplied by 2 to obtain the *annual* YTM.

DETERMINING BOND PRICES

PRACTICE: You purchase a 30-year Apple bond on the secondary market that has 6 years left to maturity and pays 8% coupons on a face value of \$5,000. You pay a price such that the bond is yielding 8.74%. Will this be a premium or discount bond? What did you pay for this bond? Given the price you compute, what is the **current yield** on this bond?

SOLUTION: The coupon is _____ than the yield to maturity. This must be a _____ bond. This means that the price we compute must be _____ than the par value of the bond.

To find the price:

N | I/Y | PV | PMT | FV

A bond's **current yield**, not to be confused with the **yield to maturity** or "**yield**", is the bond's annual coupon divided by the bond's current price. For the Apple bond above:

$$Current\ Yield = \frac{Annual\ Coupon}{Price} = \frac{\qquad\qquad}{\qquad\qquad} =$$

This tells us the annual coupon payment as a percentage of the price you pay.

Premium Bonds: Coupon Rate > Current Yield > YTM *and* Price > Par

and

Discount Bonds: Coupon Rate < Current Yield < YTM *and* Price < Par

PRACTICE: How much should you pay for a 30-year, 5% coupon Home Depot bond with a $1,000 face value that was issued 5 years ago and has 25 years left to maturity? Lowe's just issued a similar bond at par with a 7% coupon.

SOLUTION: If Lowe's, who we assume is a reasonably similar company to Home Depot with a similar credit rating just issued a reasonably similar bond in the current interest rate environment that pays 7%, we should apply that discount rate to the Home Depot bond.

N	I/Y	PV	PMT	FV

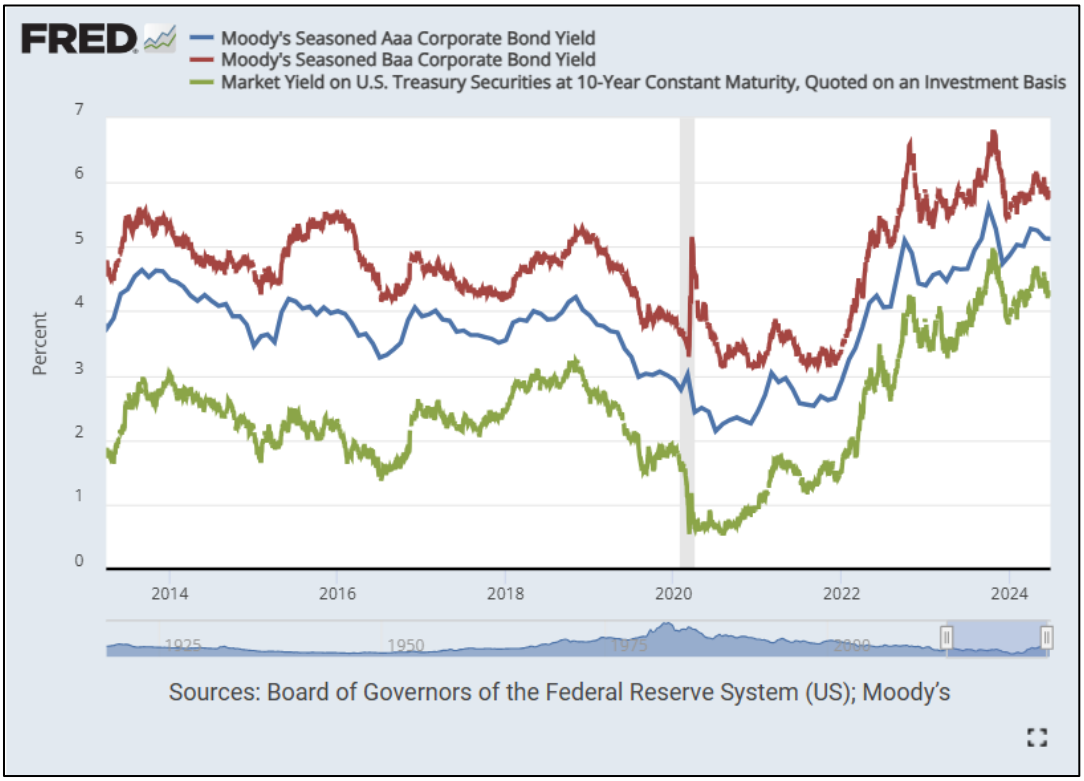
INTERPRETATION: Companies issue bonds at *prevailing market rates*. That is, if interest rates in the economy are such that investors expect a 7% return from investment grade debt, then investment grade companies will issue their new bonds with yields around

7%, often by issuing at par and paying a 7% coupon. Through time, interest rates change, but *previously* issued bonds' coupons remain the same. Therefore, the prices that investors are willing to pay for the bond in the secondary market will fluctuate such that the yield is similar to what the prevailing interest rate is in the economy.

Bonds of similar risk and structure (maturity, callability, etc.) will have prices in markets such that they yield about the same return, regardless of the coupon rate.

We can find current and historical bond yields for corporate and government issuers on the **St. Louis FED's FRED** website: https://fred.stlouisfed.org/. The existing rates give us estimates of appropriate discount rates to use in our bond valuations.

Figure 2: Bond Yields[1]



For a dynamic bond price and yield calculator, see the Excel file *Bond Price and Yield Calculator* at www.josephfarizo.com/fin360.html.

FACTORS INFLUENCING BOND YIELDS

There are four key risks influencing bond yields:

1. **Interest Rate Risk**: the risk that rising interest rates will reduce the value of the bonds you hold. For example, if you bought a 30-year $1,000 bond with 5% coupons and interest rates rise in the economy to 7%, your bond becomes less valuable than the new bonds issued with coupons nearer to 7%. If you wanted to sell your bond, you'd have to lower its price such that the buyer's yield would be close to 7%.

2. **Credit or Default Risk**: the risk that the bond issuer will be unable to pay the coupons or principal.

3. **Liquidity Risk**: the risk that your bond can't be sold quickly without a significant loss in value.

4. **Inflation Risk**: the risk that rising prices will erode the purchasing power of your bond's coupon payments. **Nominal rates** are the quoted rates of interest and return that provide actual returns in dollars, while **real rates** are the returns in terms of purchasing power, inclusive of the effects of inflation.

   The **Fisher Effect**, named after the economist Irving Fisher, shows the relationship between nominal rates R, real rates r, and the inflation rate h as:

$$1 + R = (1 + r) \times (1 + h)$$

EXAMPLE: If a bond sells as par and pays a 5% coupon, your nominal rate is 5%. If you expect inflation to be 3%, your real rate of return will be:

$$1 + R = (1 + r) \times (1 + h)$$

$$1 + 0.05 = (1 + r) \times (1 + 0.03)$$

$$1.05 = (1 + r) \times (1.03)$$

$$\frac{1.05}{1.03} = (1 + r)$$

$$\frac{1.05}{1.03} - 1 = r$$

$$r = 0.01942 = 1.942\%$$

INTERPRETATION: Your purchasing power is expected to increase by 1.942% even though the bond yields 5%. In practice, we should be careful to consider the effects of inflation, as nominal returns alone cannot guarantee an increase in (or even preservation of) purchasing power.

CRITICAL THINKING & CONCEPTUAL QUESTIONS

1. Explain how time value of money is used in the process of valuing a bond. Why isn't the value of a bond always just the par value?
2. Given coupon rates on corporate bonds are fixed, why do we care if interest rates change in the economy if we will still continue to collect our same coupon payments?
3. Explain what discount, premium, and "at par" bonds are.
4. How does the bond's current yield differ from its yield to maturity?
5. What is the relationship between a bond's coupon rate, current yield, and yield to maturity if the bond is a premium bond? What is the relationship between a bond's coupon rate, current yield, and yield to maturity if the bond is a discount bond?
6. What's the difference between a real and nominal rate of return?
7. A friend of yours is a very conservative investor. They choose to only hold 20-year government bonds because they are free of default risk and will always pay their coupons. How would you explain to your friend that holding long-term government bonds, even though they are backed by the federal government, can be very risky for their portfolio and potentially lose value?
8. Why do we use yields of other similar and newly-issued bonds if we want to determine what the value of another bond is today?
9. How and why might we adjust the discount rate we use in our bond valuation calculations?
10. Two firms want to issue bonds in the primary market to raise money. Both firms will issue senior unsecured callable debentures with 5% coupons, 30 years to maturity, and a $1,000 par value. However, one of these firms is AAA rated while the other is BBB rated. What will the BBB-rated company need to do in order for it to attract investors and sell these bonds in the primary market?
11. Describe the four key risks influencing bond yields. Why are longer-term bonds susceptible to more risk in each case?
12. How is it possible for different investors to compute different values for the same corporate bond?
13. If the FED raises interest rates, what happens to the value of existing bonds. What if the FED lowers interest rates? Why?
14. If the FED raises interest rates, what do we expect will happen to the coupon payments that new bonds pay?
15. Why does AAA-rated debt have a lower yield than BBB-rated debt? Why do government bonds yield the least? And what happens to yields as investors increase demand for and buy bonds? Why?

ANALYTICAL QUESTIONS

Determine the value (the price) of the AAA-rated corporate bond below. You will need to refer to Figure 2 to complete this problem. The figure shows that the bond's price at the time the quote was obtained on April 3, 2023, was 96.67 (or 96.67% of $1,000 = $966.70.) How does your price compare? What explains why your price is different? For simplicity, assume the bond matures in exactly 3 years.

NetAdvantage

Johnson & Johnson (NYSE:JNJ) > GLOBAL SR NT 2.950 Mar-03-2027 > Fixed Income Security Detail

Fixed Income Security Detail		Find Similar Securities	
Issuer Information			
Issuer	Johnson & Johnson (NYSE:JNJ)	Country	United States
Ultimate Parent	Johnson & Johnson (NYSE:JNJ)	CUSIP	478160CE2
Primary Industry	Pharmaceuticals	Capital IQ Trading ID	IQT421942934
Issue Summary			
Coupon At Offer	2.950	Security Type	Corporate Debentures
Coupon Type	Fixed	Seniority	Senior Unsecured
Maturity Date	Apr-03-2027	Benchmark Spread (bps)	+57.00
Offering Date	Feb-28-2017	Benchmark Security	-
Offering Price	99.897	Gross Spread	4.00
Offering Yield	2.962	Form of Ownership	Book Entry
Offering Amount ($mm)	1,000.00	Day Count Basis	30/360
Amount Outstanding ($mm)	1,000.00	Issued Currency	USD
Principal Amount	1,000.00		
Issue Features	Investment Grade, Redeemable, Callable		
Advisors			
Lead Underwriter(s)		Goldman Sachs & Co. LLC, Merrill Lynch, Pierce, Fenner & Smith Incorporated, J.P. Morgan Securities LLC, Citigroup Global Markets Inc.	
Underwriter(s)		BNP Paribas Securities Corp., HSBC Securities (USA) Inc.	
Trustee		The Bank of New York Mellon Trust Company, National Association	
Pricing and Valuation			
Evaluation Date	Apr-3-2023	Spread to Worst (bps)	25.000
Price	96.67	Current Benchmark	NT SER V-2028
Yield to Worst	3.875	Current Benchmark Maturity	Feb-29-2028
Yield to Worst Date	Mar-03-2027	Duration	3.642
Yield to Worst Date Type	Maturity	Convexity	15.500

NOTES & REFERENCES

[1] See https://fred.stlouisfed.org/seriesBeta/AAA# and "Edit Graph" to add data series for Baa and Treasuries.