University of Richmond
Robins School of Business™

# §5. Time Value of Money

**Fin 360: Principles of Financial Management**

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## INTRODUCTION TO TIME VALUE OF MONEY CONCEPTS

**EXAMPLE:** You open a bank account with $100 that pays 5% per year. How much do you have at the end of one year?

**SOLUTION:**

$$\$100 \times (1 + 0.05) = \$105 \tag{1}$$

At the end of the year, you have $105 in that account. You decide to leave it at the bank for one more year. How much will you have at the end of the second year, assuming the bank again pays 5% per year in interest?

$$\$105 \times (1 + 0.05) = \$110.25 \tag{2}$$

Substituting expression (1) into expression (2) and rewriting gives:

$$\begin{gathered} \$105 \times (1 + 0.05) = \$110.25 \\ \$100 \times (1 + 0.05) \times (1 + 0.05) = \$110.25 \\ \$100 \times (1 + 0.05) \times (1 + 0.05) = \$110.25 \\ \$100 \times (1.05)^2 = \$110.25 \end{gathered} \tag{3}$$

In the above example, the **present value (PV)** of $100 has grown to a **future value (FV)** of $110.25, given the **interest rate (r)** of 5% over two **periods (t)**. Formulating expression (3) results in:

$$\mathrm{PV}(1 + \mathrm{r})^{\mathrm{t}} = \mathrm{FV}$$

We can also algebraically rearrange this formula and solve for the **PV** by dividing through by $(1+r)^t$, giving us:

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1 + \mathrm{r})^{\mathrm{t}}}$$

This is a useful formula if we know what the FV needs to be for some spending in the future but are unsure of what the PV we should invest today is.

**EXAMPLE:** You plan on renting a new apartment in 3 years that will require a $2,000 deposit. The bank offers a safe investment that returns 6% per year. How much do you need to invest *today* to be able to withdraw $2,000 in 3 years?

**SOLUTION:** First, identify the components:

FV = $2000 r = 6% t = 3 PV = ?

Next, plug the values into the formula and solve:

$$PV = \frac{FV}{(1+r)^t}$$

$$PV = \frac{2000}{(1+0.06)^3}$$

$$PV = \frac{2000}{(1.06)^3} = \frac{2000}{1.191016} = \$1{,}679.24$$

**INTERPRETATION:** The **present value** is $1,679.24, implying you must invest this amount today to have the **future value** of $2,000 you need for the deposit.

Would you need to invest *more* or *less* today if r = 10%? What can we say about the present value as interest rates get bigger?

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These simple examples highlight a fundamental concept in finance known as the **time value of money (TVM)**. It is the idea that a sum of money today is worth more to you than that same sum of money in the future because of the ability of the money to earn a rate of return over time. Further, the certainty of having money today makes it more valuable given the possibility it may not be received at all in the future.

**EXAMPLE:** You win a raffle for $1,000. Your prize money can be collected anytime within one year. A local bank pays 5% interest per year. If you pick up the money today, you will have $1,000 + (5% of $1,000) = $1,050 in one year from today. If you wait one year, you will only get the $1,000 with an entire year of missed interest.

A dollar today is worth more than a dollar in the future.

The time value of money is the single most important concept in this course, and a central theme in finance and all of business. It has numerous real-world applications:

1. **Stock valuation**: determine what a stock is worth based on the present value of its dividends or "free cash flows" the firm generates

2. **Bond valuation**: determine what a bond is worth based on the present value of its interest payments and the principal to be paid at maturity

3. **Company valuation**: determine what a firm is worth based on the present value of its future cash flows

4. **Project valuation**: determine whether a factory should be built, or another company acquired given the cash flows it is projected to yield

5. **Personal finance**: determine how much is needed to save today for a down payment on a home or how much you need to save to retire

With the framework of the previous examples, as well as an understanding of just how important TVM is, we can dive deeper into more examples and applications.

## FUTURE VALUES AND COMPOUNDING

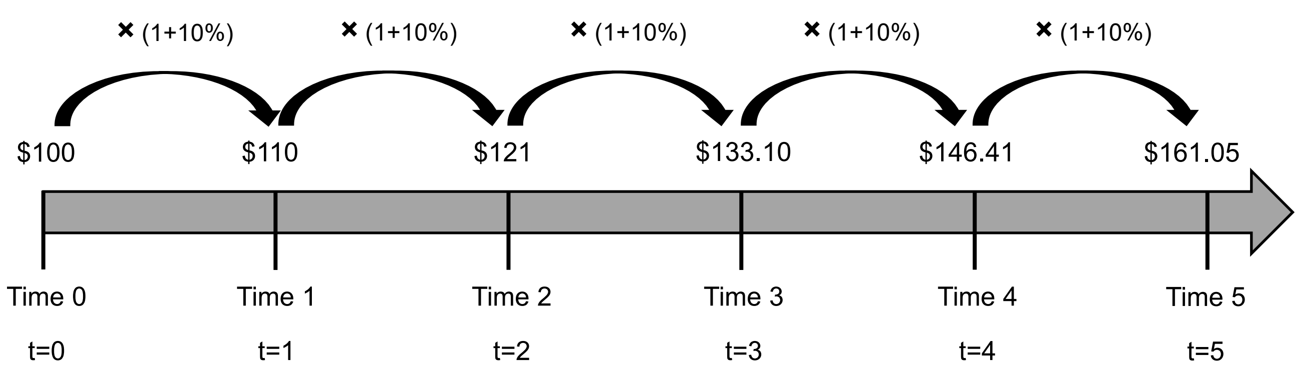
Recall from our examples above the formula:

$$FV = PV(1 + r)^t$$

This formula illustrates **compounding** or **compound interest**, which is the accumulation of money based on "*new* interest earned on interest *previously* earned." This compounding effect is evident in the $(1+r)^t$ portion of the equation (where $(1+r)^t$ is referred to as the **future value factor**).

> **EXAMPLE:** Suppose you invest $100 in an account which earns 10% per year, or interest **compounds annually**. After 5 years, we know we will have:
>
> $$FV = PV(1 + r)^t = 100(1 + 0.10)^5 = \$161.051$$
>
> We would have *less* if we earned **simple interest**, or just 10% of the original principal each year. That would result in an ending value of just $150: 100 + [(10% of 100) × 5] = $150.

*Figure 1: Future Value Timeline*




Notice how compounding allows you to earn additional interest on top of both the **principal** amount invested (the original $100), and the **simple interest** earned in the previous period.

We can also write (as we have seen before):

$$FV = \$100 \times 1.10 \times 1.10 \times 1.10 \times 1.10 \times 1.10$$

$$FV = \$100 \times (1.10)^5$$

$$FV = \$161.05$$

Compounding is powerful, particularly as we increase the number of periods $t$. Consider the figure below which demonstrates the "interest earned on interest" for the previous example:

*Table 1: The Power of Compounding*

| I. Year | II. Beginning amount | III. Simple Interest (10% of principal) | IV. Compound Interest (10% on interest earned) | V. Total Interest Earned (III + IV) | VI. Ending Amount (II + V) |
|---|---|---|---|---|---|
| **1** | \$100 (principal) | \$10 | \$0 | \$10 | \$110 |
| **2** | 110 | 10 | 1 | 11 | 121 |
| **3** | 121 | 10 | 2.1 | 12.10 | 133.10 |
| **4** | 133.10 | 10 | 3.31 | 13.31 | 146.41 |
| **5** | 146.41 | 10 | 4.64 | 14.64 | 161.05 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| **50** | 10,671.90 | 10 | 1057.19 | 1067.19 | 11,739.09 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| **100** | 1,252,783 | 10 | 125,268.3 | 125,278.3 | 1,378,061 |

? How much would the 10% investment above grow to after 50 years of simple interest? With compound interest? How does this difference change as years pass?

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> Crucially, banks almost always pay compound interest. Investments in the stock and bond markets grow with compounding. Balances on debts like mortgages and credit cards also experience compounding – balances are carried from one period to the next!

## PRESENT VALUES AND DISCOUNTING

Revisiting the present value formula,

$$PV = \frac{FV}{(1 + r)^t}$$

we can find what some value in the future is worth – *today*. In this context, *r* is referred to as the **discount rate**. Functionally, it is the same as the *r* in the future value formula, but the terminology changes. The $1/(1+r)^t$ portion of the equation is referred to as the **present value interest factor**.

*Table 2: Interest Rates*

| Terminology for *r* | | |
|---|---|---|
| Convert from PV to FV | Interest rate, growth rate | Growing, compound growth, compounding |
| Convert from FV to PV | Interest rate, discount rate | Discounting |

**EXAMPLE:** Suppose you wish to have $161.05 in an account (that earns 10% per year) in 5 years. How much do you need to invest today?

$$PV = \frac{FV}{(1 + r)^t} = \frac{\$161.05}{(1 + 0.10)^5}$$

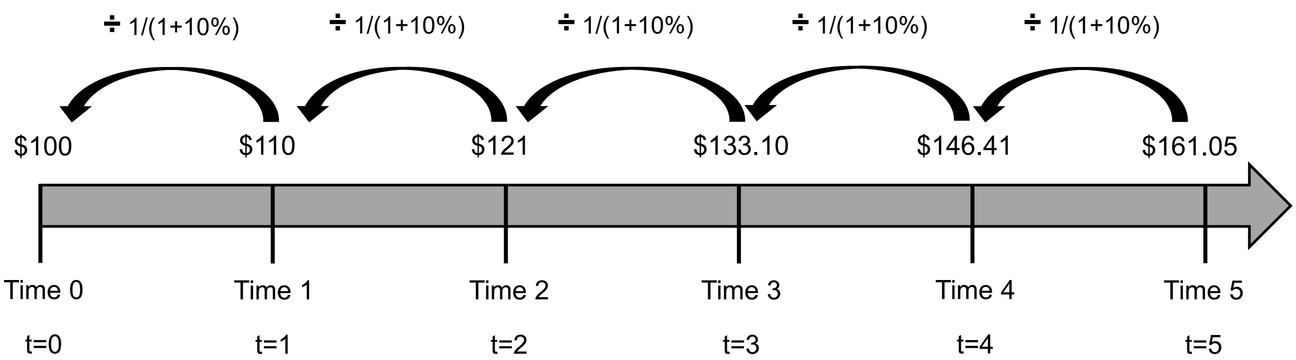
We can also write:

$$PV = \frac{\$161.05}{1.10 \times 1.10 \times 1.10 \times 1.10 \times 1.10}$$

$$PV = \frac{\$161.05}{(1.10)^5}$$

$$PV = \$100$$

*Figure 3: Present Value Timeline*




## Applications of TVM

In practice, using a financial calculator (or Excel and other software) is common. First, we will need to set up our calculator, the TI-BA II Plus.

### Using a Financial Calculator

1. Display the maximum number of decimal places:

   **2ND** → **FORMAT** → use ↑ ↓ arrows to navigate to **DEC** → **9** → **ENTER**

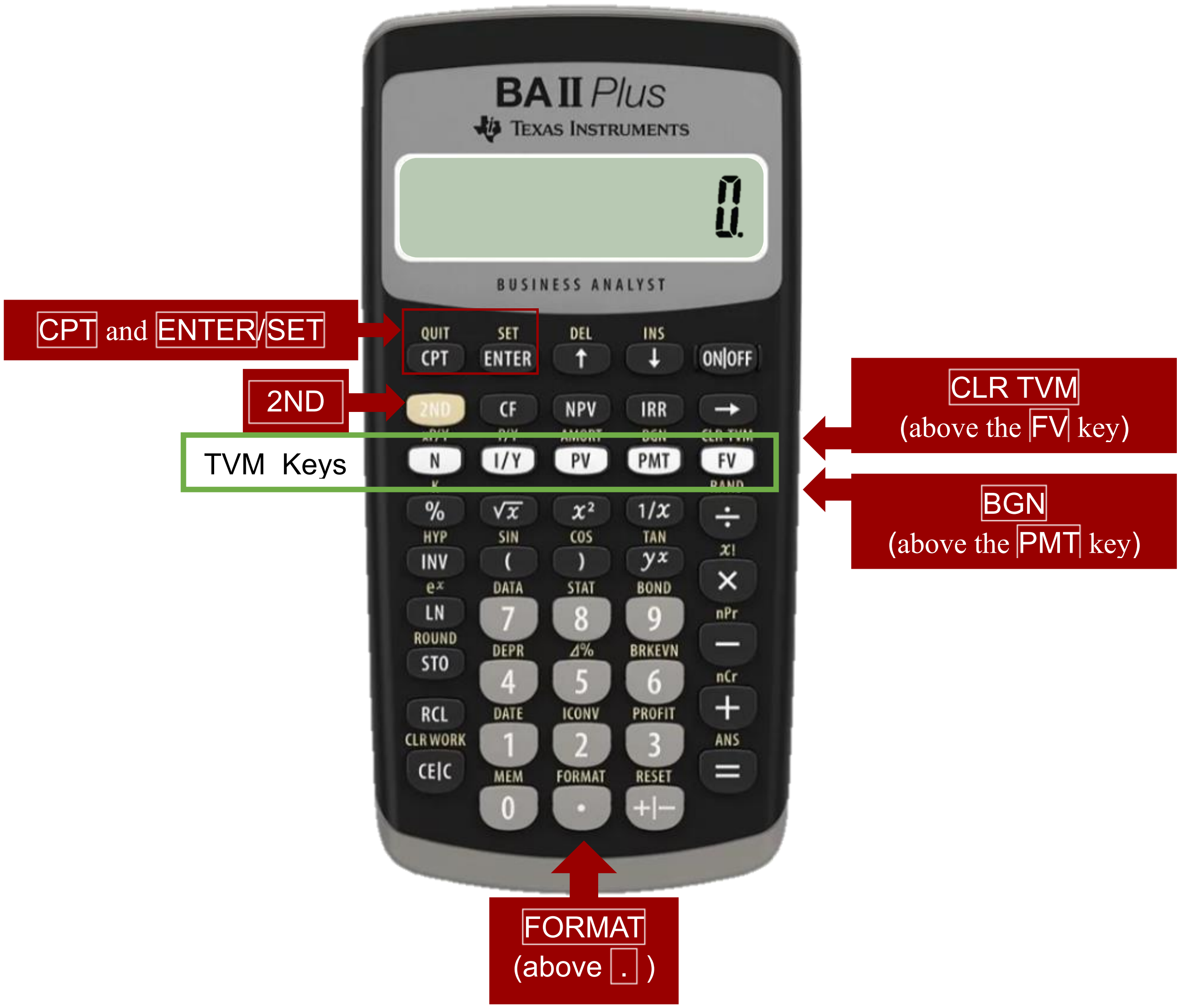
2. Make sure there is no "BGN" above the zero on your screen:

   **2ND** → **BGN** → **2ND** → **SET**

3. Clear the calculator's TVM memory:

   **2ND** → **CLR TVM**

*Figure 4: TI BAII Plus Calculator*




You must clear the stored values in the TVM keys before *every* TVM problem. Failure to do so can result in incorrect answers or error messages.

**PRACTICE:** A firm estimates that 17 key employees will be retiring in 8 years, and each employee is due a retirement gift of $10,000 cash. The firm can invest money today in an account earning 4% annually. How much do they need to put in the account today to be able to fund these bonuses?

**SOLUTION:** This is a PV problem, with the following information given:

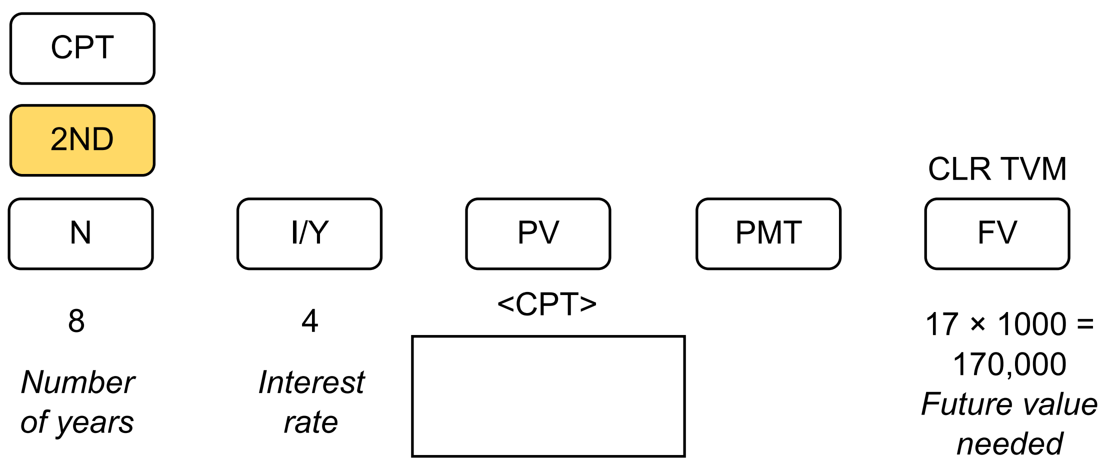
FV = × =

$r$ or I/Y =

$t$ or N (<u>N</u>umber of periods) =

PV = ?

We perform the following keystrokes (pressing the number first then the relevant key):




Which matches the formula:

$$\text{PV} = \frac{\$10{,}000 \times 17}{(1 + 0.04)^8} = \$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

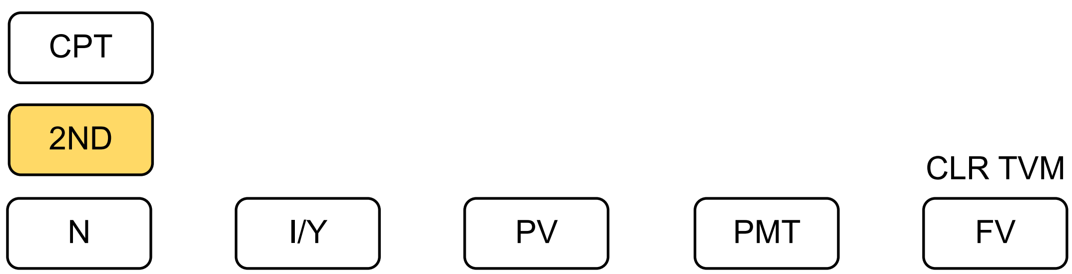
The calculator gives a negative present value. This is because the firm must invest that money (an outflow) to receive the future value (an inflow) in the future. Of course, the "true answer" is positive. The sign is just the calculator telling us the direction of the cash flow. We do not use the negative sign in the formula.

**INTERPRETATION:** The firm must set aside $124,217.33 in order to have the $170,000 needed in 8 years, assuming that money can grow at 4% annually.

## ADDITIONAL EXAMPLES

**PRACTICE:** A firm plans to pay $13 million in cash to begin the construction of a factory in 4 years. They have identified an investment opportunity that will yield them 8% annually in the meantime. (a) If the firm invests $9 million today, will they be able to afford the $13 million in 4 years? (b) Exactly how much would they need to invest today in order to have the $13 million when it is needed?

**SOLUTION:** This is a two part problem, highlighting both a present value and future value approach. In part (a), we know the firm has $9 million today, and we need to determine what that will be worth in 4 years.




By the formula, we get the same answer:

$$FV = PV(1 + r)^t = \qquad (\ 1\ +\qquad )^{4} \ =$$

Would the firm be able to afford the $13 million in 4 years?

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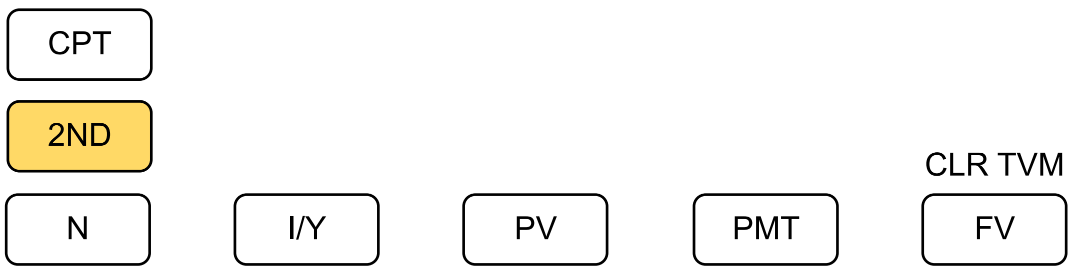
In part (b), we'll determine the exact amount the firm would need to invest today in order to have the $13M in 4 years. This is a present value problem: we know what they need in 4 years, so we need to figure out how much they need to invest in the present.




By the formula, we get the same answer:

$$PV = \frac{FV}{(1+r)^t} = \frac{\qquad}{(1+\qquad)} =$$

Exactly how much will the firm need to invest today (at the 8% interest rate) to be able to afford the $13 million in 4 years?

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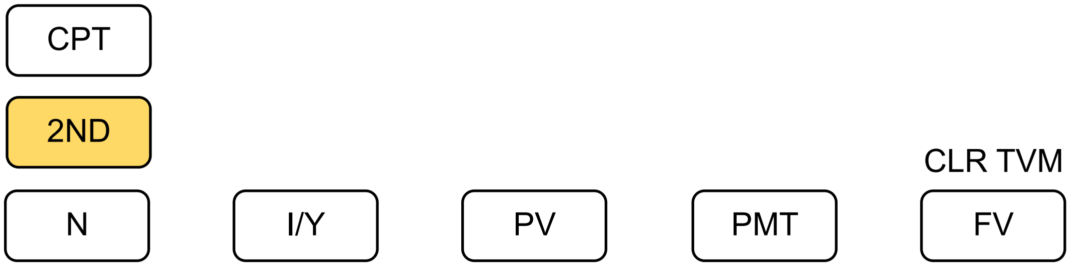
## Present Values in the Future

**Practice:** How much do you need to invest in 3 years if you plan on having $30,000 in 10 years in order to purchase your dream motorcycle, the Harley-Davidson Street Glide, assuming your investment will be able to grow at 12% per year?

**SOLUTION:**




**INTERPRETATION:** Be mindful of the terms "present value" and "future value". The "present value" doesn't necessarily mean today. It is technically defined by the formula: a function of the future value, interest rate, and number of periods.

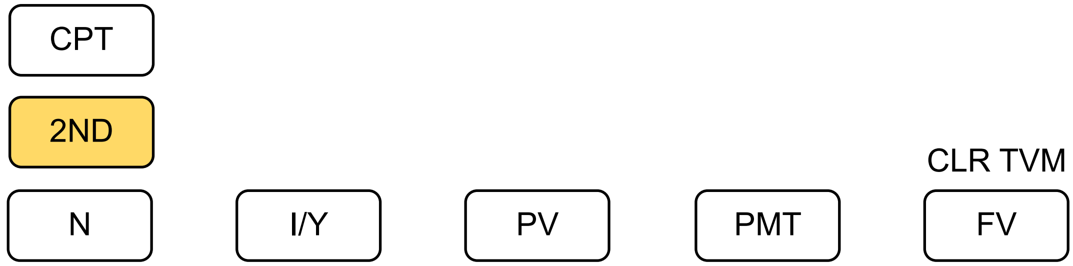
> "Future (present) values" do not need to be in the "future (present)." These terms just tell us where we are *relative* to the other present or future values.

## SOLVING FOR OTHER INPUTS

The formulas for present value and future value imply that we might solve for the time periods N and the interest rate I/Y when we know the present value and future values.

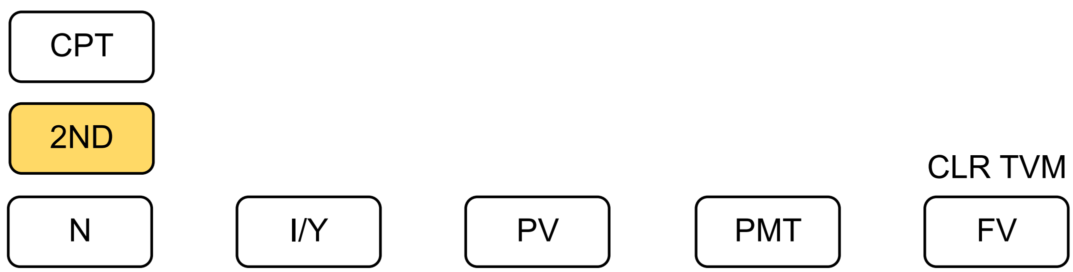
**PRACTICE:** In 3 years you expect to begin a new career, and your employer will give you a signing bonus of 10% of your $78,000 base salary. At that time, you plan on investing that bonus at 3.8917% per year until you have $12,000, enough for you to take a 11-month backpacking trip through Europe. In how many years from now will you be able to afford this trip? What would the interest rate need to be if you needed $13,000 instead of $12,000?

**SOLUTION:** Here, we are looking for the number of years from *now* where we will be able to afford this trip.




Is the computed "N" the answer to the question?

How much would the interest need to be if you instead needed $13,000 for the trip?




Additional TVM examples are available in the Excel file *TVM Practice Problems* at www.josephfarizo.com/fin360.html.

## Critical Thinking & Conceptual Questions

1. Why is a dollar to be received one year from now worth less than a dollar today? Why is a dollar in hand worth more than a dollar to be received in one year?
2. Discuss how time value of money concepts are necessary for the 5 broad "real-world applications" we've talked about in class and in the lecture notes.
3. Explain in words and with an equation the relationship between present and future values.
4. Describe "compounding" and "discounting." Give an example of an application of each.
5. Explain how an investment of $100 in the stock market pays compound returns.
6. Describe what a "present value interest factor" is and how it is obtained.
7. Describe what a "future value interest factor" is and how it is obtained.
8. You have two friends (Luke and Kylo) who each stand to inherit $10,000 in exactly 1 year. Both friends, however, need money now. They promise to give you $10,000 when their inheritance check clears in exactly one year, if you give them some money today. Luke is organized, trustworthy, and reliable. Kylo is disorganized and not always dependable.
    a. Would you give more money to Luke or Kylo today and why? Why not give them each the same amount?
    b. To whom will you assign the higher discount rate and why?
    c. If you consider giving money to Luke and Kylo today an "investment," which of your "investments" should offer a greater rate of return?
    d. Explain how questions (b) and (c) above are related.
9. You identify two small firms with the same SIC code (Republic Inc. and Empire Inc.) that each will realize a cash flow of $10,000 in exactly 1 year. Both firms, however, need money now. They promise to give you $10,000 when their cash flows are realized in exactly one year if you give them some money today. Republic has a low equity multiplier, a high cash coverage ratio, and a high current ratio. Empire has a high debt to equity ratio, a low cash coverage ratio, and a low current ratio.
    a. Would you give more money to Republic or Empire today and why? Why not give them each the same amount?
    b. To which firm will you assign the higher discount rate and why?
    c. If you consider giving money to Republic and Empire today an "investment," which of your "investments" should offer a greater rate of return?
    d. Explain how questions (b) and (c) above are related.
10. Describe what happens to the difference between simple interest and compound interest as we increase the number of periods $t$ that we hold an investment.
11. Explain how a present value can be in the future. Explain how a future value can be in the past.
12. When computing present and future values in our calculator, the results are often presented with a negative sign. Explain why this negative sign appears, and why the results of our computation are not "actually" negative.
13. Lotteries often give winners the option of taking either a "lump sum" or a series of payments over several years. For example, if you win $1 million, you can either take all $ 1 million now, or receive $100,000 for the next ten years. Assuming no taxes, which would you choose and why?

## ANALYTICAL QUESTIONS

1. Given what you know about the formula for present value interest factors, fill in the missing values in the table below:

| Present Value Interest Factor Table | | | | |
|---|---|---|---|---|
| Years | 1% | 2% | 3% | 4% |
| 1 | 0.990099 | 0.980392 | 0.970874 | 0.961538 |
| 2 | 0.980296 | 0.961169 | | 0.924556 |
| 3 | 0.97059 | | 0.915142 | 0.888996 |

2. Given what you know about future value interest factors, fill in the missing value in the table below:

| Future Value Interest Factor Table | | | | |
|---|---|---|---|---|
| Years | 1% | 2% | 3% | 4% |
| 1 | 1.01000 | | 1.030000 | 1.040000 |
| 2 | 1.02010 | 1.040400 | 1.060900 | 1.081600 |
| 3 | 1.030301 | 1.061208 | 1.092727 | |

3. Using the tables above, determine the present value of $100 received in 2 years, discounted at 3%. Determine the future value of $100 after 3 years assuming 3% interest. Confirm both calculations in your calculator.