University of Richmond
Robins School of Business™

§6. Discounted Cash Flow Valuation

Fin 360: Principles of Financial Management

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Multiple Cash Flows

In practice, firms and individuals might receive, invest, or pay a series of cash flows rather than receive, invest, or pay one **lump sum**. We now consider the future and present values of multiple cash flows over a period of time. Note that the intuition remains the same: we will continue to use the future value and present value formulas.

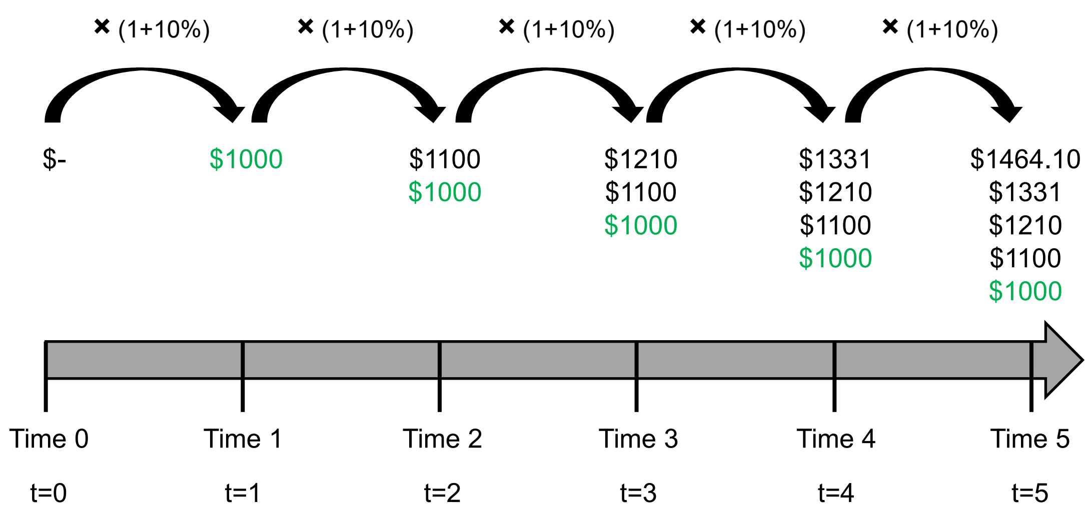
Future Value of Multiple Cash Flows

Determining the future value of multiple cash flows is useful for calculating, for example, how much you will have in an account after years of saving.

Example: You open a bank account that earns 10% annually. Beginning at the end of the year and then on the last day of each of the next 5 years, you deposit $1,000 into this account. How much will you have at the end of 5 years?

Solution: Here, it is important to visualize with a timeline. In green is the new deposit to the account made at the end of each period.

Figure 1: Future Value Multiple Cash Flow



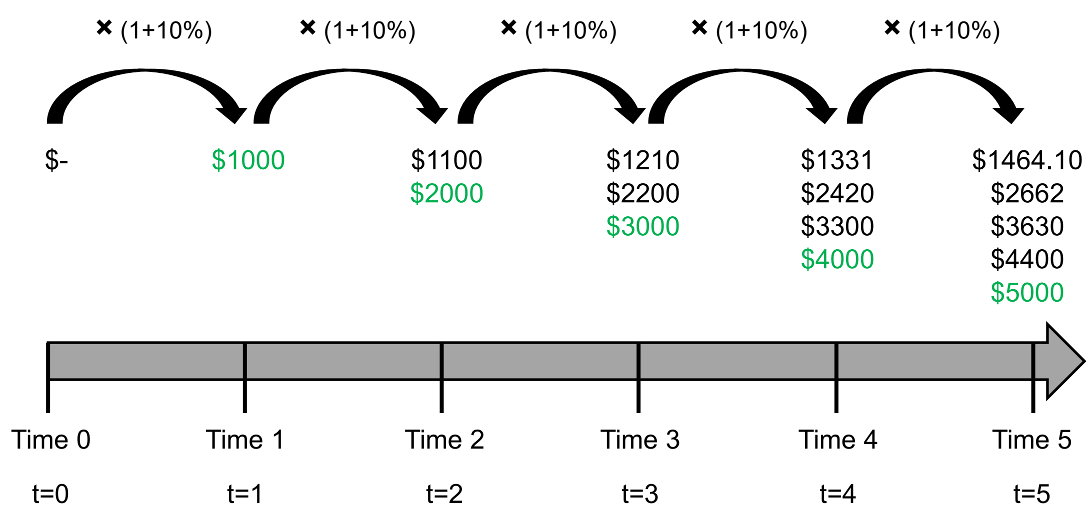
At the end of 5 years, there is $1464.10 + 1331 + 1210 + 1100 + 1000 = **$6,105.10** in the account. Using the FV formula, you could also write this as:

$$(1000 \times 1.1^4) + (1000 \times 1.1^3) + (1000 \times 1.1^2) + (1000 \times 1.1^1) + (1000 \times 1.1^0)$$

whereby each $1,000 deposit *compounds* its relevant number of periods. Note that because you waited a year to make your first payment, the initial $1,000 only compounds 4 times.

The deposit amounts do not need to be constant. Assume now that each year you increase the deposited amount by $1,000 (i.e., deposit $1,000 then $2,000 then $3,000...). The timeline and formula would become:

Figure 2: Future Value Multiple Growing Cash Flows (end of period)

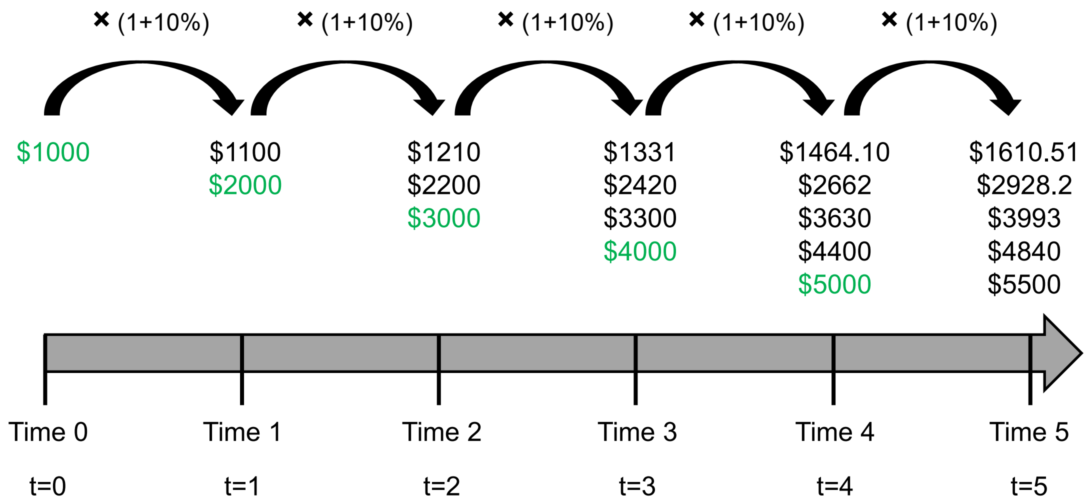


$$(1000 \times 1.1^4) + (2000 \times 1.1^3) + (3000 \times 1.1^2) + (4000 \times 1.1^1) + (5000 \times 1.1^0)$$

The future value in this case is **$17,156.10**. Note that the timing of the cash flow (at the beginning or end of the period) matters.

If you instead deposit $1,000 dollars today, then make your deposits at the beginning of each year:

Figure 3: Future Value Multiple Growing Cash Flows (beginning of period)



In our formula, notice the exponents are "stepped up" as each deposit compounds for an additional period:

$$(1000 \times 1.1^5) + (2000 \times 1.1^4) + (3000 \times 1.1^3) + (4000 \times 1.1^2) + (5000 \times 1.1^1)$$

The future value here is **$18,871.71**. Despite contributing the same $15,000 to the account, you have $18,871.71 – $17,156.10 = $1,715.61 more in this case because of the additional year of compounding.

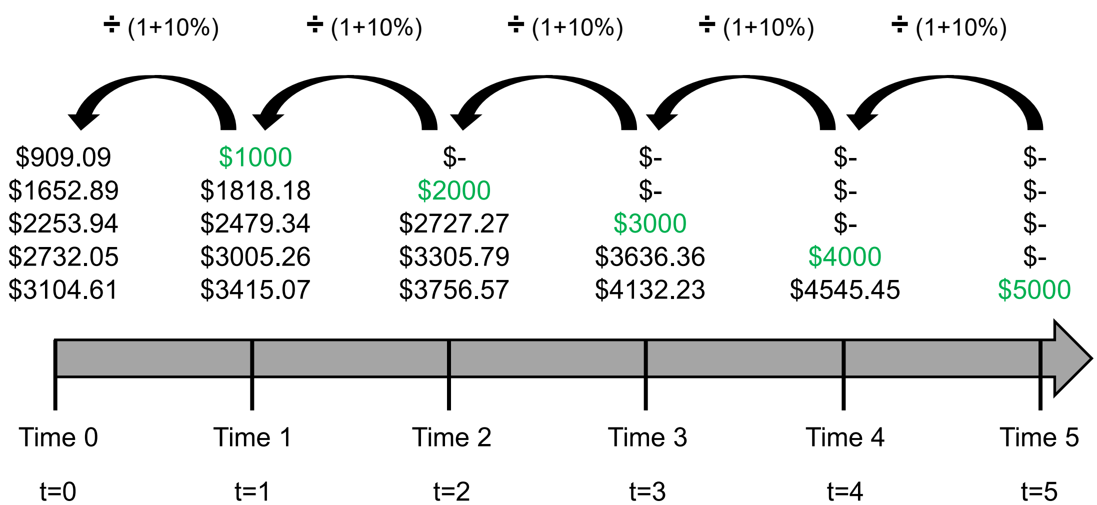
Present Value of Multiple Cash Flows

Similarly, we can find the present value of multiple cash flows, or what a series of cash payments to be received in the future are worth today. This is the foundation of stock, bond, and company valuation since each "generates" multiple cash flows through time. We call this process and similar applications **discounting cash flows**.

EXAMPLE: What would a series of deposits of $1000, $2000, $3000, $4000, and $5000 made into an account at the end of each of the next 5 years be worth today? Assume a 10% annual discount rate.

SOLUTION: Again, visualizing with a timeline helps. The deposits are in green. Notice that we now divide by (1+10%) for each period, consistent with the present value formula.

Figure 4: Present Value Multiple Growing Cash Flows (end of period)



Therefore, the series of deposits made at the end of each year ($1000, $2000, $3000, $4000, and $5000) over the next five years is worth:

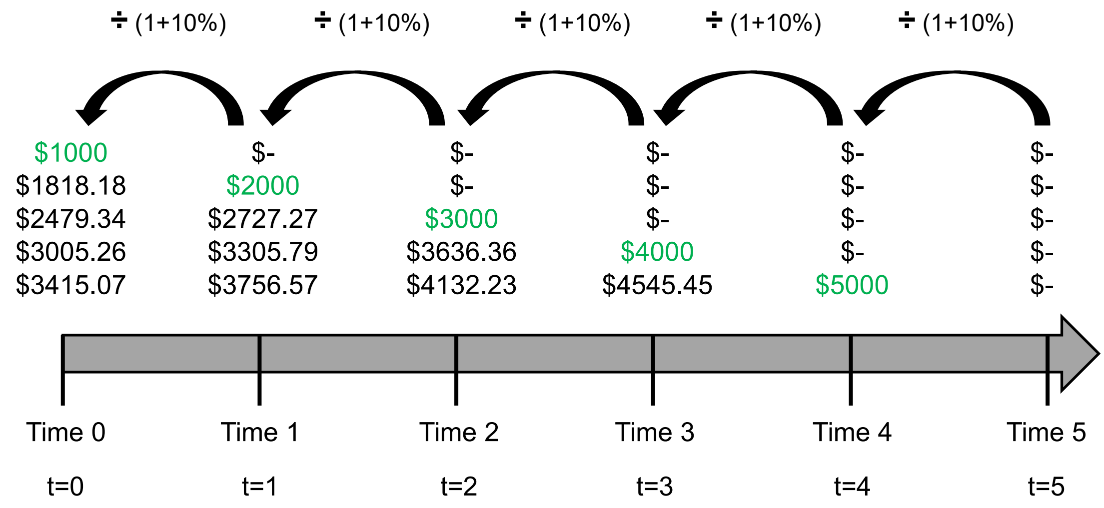
$$909.09 + 1652.89 + 2253.94 + 2732.05 + 3104.61 = \mathbf{10,652.58}$$

today assuming a discount rate of 10%. By the present value formula:

$$\frac{1000}{(1+0.10)^1} + \frac{2000}{(1+0.10)^2} + \frac{3000}{(1+0.10)^3} + \frac{4000}{(1+0.10)^4} + \frac{5000}{(1+0.10)^5}$$

which gives the same answer. If the deposits are made at the *beginning* of the year:

Figure 5: Present Value Multiple Growing Cash Flows
(beginning of period)



This results in a present value of **$11,717.85**. This is greater than in the end-of-period case (PV = **$10,652.58**), given the deposits are closer to the present (and a dollar today is worth more than a dollar in the future).

Applications of Multiple Cash Flow

We can use our understanding of present and future values of cash flows to address topics such as structured settlements, lotteries, and investments in the stock market.

Practice: You are the winner of a lawsuit that entitles you to collect $100,000 a year for the next 30 years, beginning one year from today. Assume that you can invest the money each year in an account that earns 9% per year.

(1) How much will you have in the account at the end of 30 years?
(2) How much would you have in the account if you received the first payment today with future payments one year from today and every year thereafter for the 30 years?

J.G. Wentworth and Oasis Financial are financial services firms that purchase structured settlements from winners of lawsuits for a price upfront.[1] They'd pay you a lump sum amount today, and you transfer all future proceeds of the lawsuit over to them.

(3) Why would you consider selling these cash flows?
(4) Why would a firm offer to buy these cash flows?
(5) How much would you be willing to sell your payments in both (1) and (2)?

SOLUTION: First, we recognize that this is a future value question with many periods, and draw a timeline to help visualize the timing and frequency of payments:

And the formula is:

$$(100k \times \qquad\qquad\qquad) + (100k \times \qquad\qquad\qquad) + \cdots + (100k \times \qquad\qquad\qquad)$$

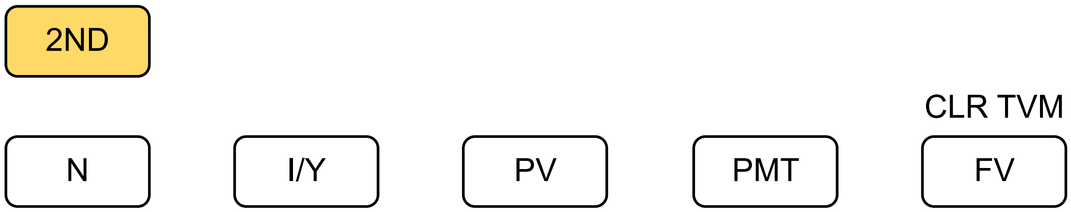
As we did above, we can compute the future value by adding up all 30 of these individual sums. Fortunately, we have a formula that we can use for fixed or constant payments:

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right]$$

Here, this would be:

$$FV = \qquad\qquad\qquad \left[\frac{(1 + \qquad\qquad\qquad)\quad - 1}{\qquad}\right] =$$

The calculator can also compute this value, by using the **PMT** key:



If the payments are at the beginning of the period, the timeline becomes:

And the formula is:

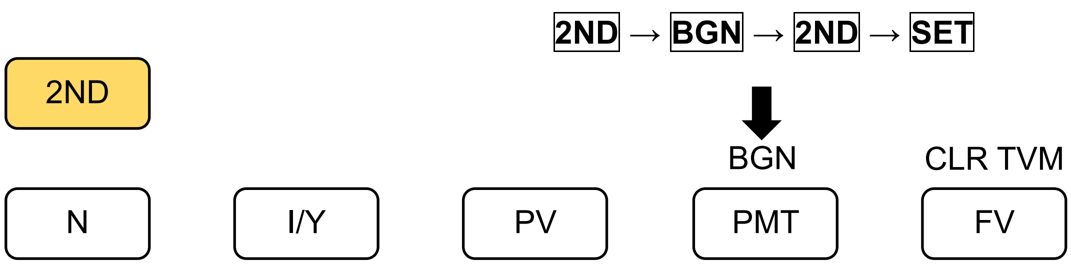
$$(100k \times \qquad\qquad) + (100k \times \qquad\qquad) + \cdots + (100k \times \qquad\qquad)$$

We also modify the formula by multiplying the term *(1+r)* since the payments have an additional compounding period:

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right] \times (1+r)$$

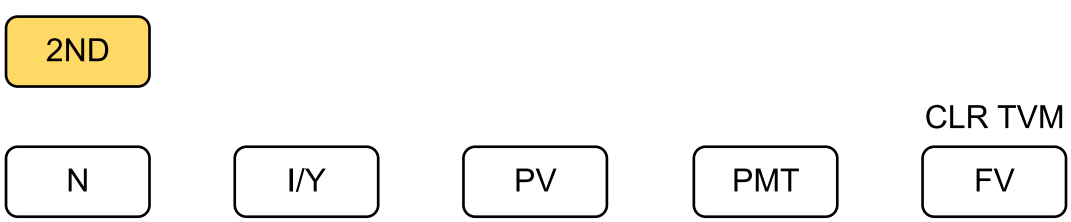
$$FV = \left[\frac{(1+\qquad)\qquad -1}{\qquad}\right] \times (1+\qquad) =$$

The calculator inputs are:

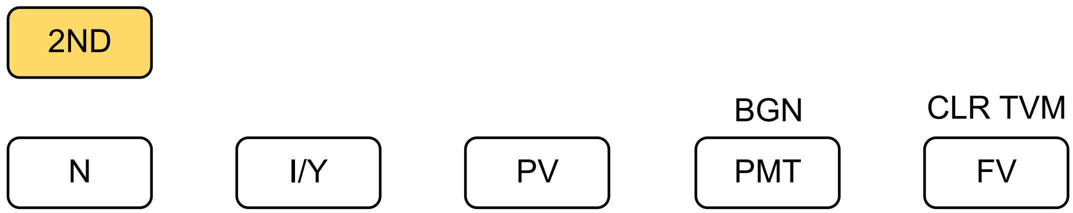


⚠ Be careful to check that your calculator is in the correct mode, only showing BGN when payments are at the beginning of the period.

This gives the same answer as the mathematical expression above. To determine what the end-of-period series of cash flows is worth for you to sell them to a financial services firm, we compute the present value with the inputs:



If payments are received at the beginning of the period:



INTERPRETATION: To summarize:

FV end-of-period payments	
FV beginning-of-period payments	
PV end-of-period payments	
PV beginning-of-period payments	

You should only sell your cash flows if you are offered at least _______________ if payments are made at the end of the year, or _________________ if payments are at the beginning of the year.

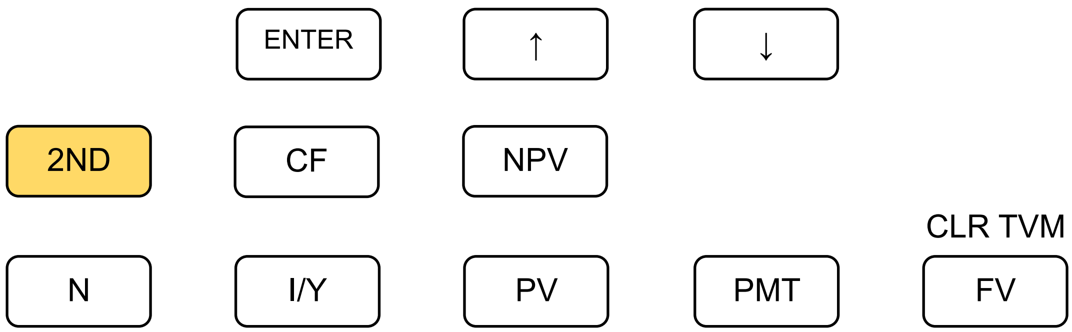
PRACTICE: An investor makes the following annual investments in the stock market:

Year 1	$2,000
Year 2	$3,000
Year 3	$4,000
Year 4	$4,000

What are these cash flows worth today? Why can't we just add up the annual values to determine what they are worth today? How much will the investor have in this account after 4 years? To address these questions, assume first investments are made (1) at the end of the years and then (2) at the beginning of the years. Assume the stock market returns 7.5% per year.

SOLUTION: Here, we have **uneven cash flows**. Because the payments vary, we cannot use the **PMT** key on the calculator, but we can use the calculator's built-in cash flow worksheet:

Assuming end-of-period cash flows (1):



Keystrokes	**Explanation**
CF	The "cash flow" key, ***CFo=*** appears on the screen
2ND **CLR WORK**	"Clear worksheet" (above **CE|C** key)
CFo = 0 Enter ↓	Cash flow at time 0: zero, payments are at the end
C01 = -2000 Enter ↓	Cash flow at time 1: -$2000, a cash outflow
F01 = 1 Enter ↓	Frequency of the first cash flow: once
C02 = -3000 Enter ↓	Cash flow at time 2: -$3000
F02 = 1 Enter ↓	Frequency of the second cash flow: once
C03 = -4000 Enter ↓	Cash flow at time 3: -$4000
F03 = 2 Enter	Frequency of the third cash flow: twice
NPV	"Net Present Value" button
I = 7.5 Enter ↓	The interest rate
CPT NPV	Compute the "Net *Present* Value"

The present value is $ ______________________, assuming end-of-period cash flows.

To find the future value of these cash flows, **CE|C** **2ND** **CLR TVM** then enter the following in the TVM keys:

PV = **N** = **I/Y** = **CPT** **FV**

The future value is $ ______________________, assuming end-of-period cash flows.

Assuming beginning-of-period cash flows (2):

<u>Keystrokes</u>	**<u>Explanation</u>**
CF	The "cash flow" key, ***CFo=*** appears on the screen
2ND **CLR WORK**	"Clear worksheet" (above **CE|C** key)
CFo = -2000 **Enter** ↓	Cash (out)flow at time 0: -$2000, payments at beginning
C01 = -3000 **Enter** ↓	Cash flow at time 1: -$3000
F01 = 1 **Enter** ↓	Frequency of the first cash flow: once
C02 = -4000 **Enter** ↓	Cash flow at time 2: -$4000
F02 = 2 **Enter** ↓	Frequency of the second cash flow: twice
NPV	"Net Present Value" button
I = 7.5 **Enter** ↓	The interest rate
CPT **NPV**	Compute the "Net *Present* Value"

The present value is $ ______________________, assuming beginning-of-period cash flows.

To find the future value of these cash flows, **CE|C** **2ND** **CLR TVM** and **2ND** **BEG** **2ND** **SET** to make sure the payments are set to **BGN**. Then enter the following in the TVM keys:

PV = **N** = **I/Y** = **CPT** **FV**

The future value is $ ______________________, assuming beginning-of-period cash flows.

INTERPRETATION: To summarize, we not that the present values and future values are both greater with beginning-of-period cash flows than end-of-period cash flows:

FV end-of-period payments	
FV beginning-of-period payments	
PV end-of-period payments	
PV beginning-of-period payments	

ANNUITIES AND PERPETUITIES

ORDINARY ANNUITIES AND ANNUITIES DUE

Annuities are streams of cash flows that are fixed amounts (i.e., $100 every year) or grow at a fixed rate (i.e., 2% per year: $100, then $102, then $104.04) over a period of time. When the payments are at the end of the period, it is called an **ordinary annuity**. If the payments are at the beginning of the period, it is called an **annuity due**.

We've seen how to calculate the PV of "fixed amount" annuities with the **PMT** key above. But what if the payments are growing at a "fixed rate"? For an **ordinary annuity**, we can use the formula:

$$Growing\ Annuity\ PV = C \times \left[\frac{1-\left(\frac{1+g}{1+r}\right)^{t}}{r-g}\right]$$

where

C = the first payment received one period from now

r = the discount rate

g = the fixed growth rate of the payments

t = the number of periods

To convert the results of the *Growing Annuity PV* formula into an **annuity due**, we apply the following formula:

$$Annuity\ Due\ PV = Ordinary\ Annuity\ Value \times (1 + r)$$

PRACTICE: Your firm has started a renewable energy project, and you've secured a government contract to receive annual subsidies for the next 20 years. The first payment is $200,000, which you will receive one year from now. Every year thereafter, your payment will grow by 5% per year to support inflation and project expansion. What is the present value if you discount these cash flows by 11%? What if the first subsidy payment is received today, with all other payments received in one-year intervals?

SOLUTION: First, we determine the present value of the **ordinary annuity**. By the formula

$$Growing\ Annuity\ PV = C \times \left[\frac{1 - \left(\frac{1+g}{1+r}\right)^t}{r - g}\right]$$

For this example, we have:

$$Growing\ Annuity\ PV = \qquad\qquad \times \left[\frac{1 - \left(\frac{1+\qquad}{1+\qquad}\right)}{\qquad - \qquad}\right]$$

$$Growing\ Annuity\ PV =$$

If payments are received beginning today, we know the present value of this sum is:

$$Annuity\ Due\ PV = Ordinary\ Annuity\ Value \times (1 + r)$$

$$Annuity\ Due\ PV = \qquad \times (1+ \qquad)$$

$$Annuity\ Due\ PV =$$

INTERPRETATION: If we add up all the payments, beginning with $200,000 then $210,000 and so on as they grow at 5% per year, the sum would be about $6.6 million. Yet, our present values are much less than that. Remember, the guiding principle of TVM is that a dollar today is worth more than a dollar in the future. Even though our 20th payment may have grown to over $505,000, it is 20 years away and equivalent to a much lower *present value* today.

> From the Oxford English Dictionary, a definition of "discount" is "to disregard or rule out as unreliable, false, or irrelevant. Also: to give little credence to."
>
> This is what we are doing when determining the present value of future cash flows. Given uncertainty and risk, we "discount" whether future cash flows will even be earned, and we "penalize" these cash flows for occurring in the future given we can't earn interest on them in the meantime.

PRACTICE: A client purchased an 8-year annuity from an insurance company for $57,726. The insurance company offers an annual rate of return of 9.25%. What are the end-of-year payments the insurance company must pay to the client?

SOLUTION: Using a formula, we can solve for the payment *PMT* algebraically:

$$PV = PMT\left[\frac{1-(1+r)^{-t}}{r}\right]$$

Alternatively, we can use the financial calculator:

2ND

CLR TVM

N | I/Y | PV | PMT | FV

INTERPRETATION: The insurance company must make payments of \$10,526.72 to deliver on the 9.25% rate of return promised to the client.

Given this understanding of a discount rate as a measure of uncertainty, do we think "riskier" cash flows have higher or lower discount rates than less risky cash flows?

____

____

____

PERPETUITIES

A **perpetuity** is an annuity where the cash flows continue forever, or *perpetually*. To obtain the present value of a series of infinite and constant cash flows, we follow the formula:

$$PV_{perp} = \frac{C}{r}$$

PRACTICE: **Preferred stock**, commonly issued by banks and financial institutions, can pay a constant dividend to the shareholder every period, and is the "first" residual claimant, over other **common stock** shareholders. How much would you be willing to pay for a preferred share if it pays a \$4 dividend annually, and you think this firm should offer a return of 10% given its level of risk?

SOLUTION:

$$PV_{perp} = \frac{C}{r} = \frac{\hspace{3cm}}{} =$$

INTERPRETATION: We might choose that 10% "required return" because other similar risky firms offer that rate. If the firm is riskier, we might require a higher return (and thus be willing to pay less for the stock). If less risky, we would be content to accept a lower rate of return (and this be willing to pay more for the stock).

If the cash flows associated with a perpetuity grow at a constant rate *g*, we use:

$$PV_{growing\ perp} = \frac{C}{r - g}$$

PRACTICE: **Dividend aristocrats** are mature firms that have a long history (at least 25 years) of paying and steadily increasing their dividends. Suppose a firm announces it plans to pay a $0.25 dividend, and that it will grow that dividend at 3% each year, forever. How much should an investor pay for this stock if they require a return of 5% (given its risk) on this company's stock?

SOLUTION:

$$PV_{perp} = \frac{C}{r - g} = \frac{\hspace{3cm}}{} =$$

INTERPRETATION: Here, we have two "rates": the required rate of return and the growth rate of the dividends. It is important that we understand the distinction between these two. The required rate of return should exceed the growth rate (r > g), otherwise the growth would not be enough for the investor's required return (which they establish given the risk associated with the investment.)

As of 2024, there are about 60 dividend aristocrats in the S&P 500. There are around 12 **dividend monarchs** (or **kings**) that have raised their dividends for 50 or more years. You can learn more about dividend aristocrats on S&P Global's website.[2]

IN SUMMARY

Here, we introduced the concept of "multiple cash flow" and:

- Computed present and future values of multiple cash flows over many periods
- Discussed perpetuities and annuities, and the difference between the two
- Examined the effects of end-of-period (ordinary) and beginning-of-period (due) payments.

Soon, we will use discounted cash flow valuation to determine the value of stocks, bonds, and projects.

Additional examples are available in the Excel file: *Discounted Cash Flow* at www.josephfarizo.com/fin360.html.

Critical Thinking & Conceptual Questions

1. Explain why it would be better to be paid your entire annual salary on the first day of the year instead of receiving 26 bi-weekly paychecks throughout the year. Show with a hypothetical example.
2. Explain why beginning of period payments increase *both* the present value and future value of cash flows.
3. Explain how a series of cash flows, such as cash flows from a legal settlement, can be "purchased" or "sold" to another party.
4. What happens to the (i) present value and (ii) future value of a series of cash flows if (holding all else constant):
    a. The size of the cash flows through time increase
    b. The discount rate or compounding rate r increases
    c. The number of cash flow payments increases
    d. The payments shift from the end of the period to the beginning of period
    e. The growth rate g increases
5. How do we develop a discount rate for a series of cash flows?
6. In what way do annuities and perpetuities differ?
7. Why can't we compute the future value of a perpetuity?
8. Is a preferred stock worth *more* or *less* today if you were to increase the rate of return you require on the preferred stock, given the stock's risk.
9. In general, how do we "assign" a "required rate of return" on a stock? Explain what factors might go into your own personal decision as to what an investment should return in order for you to invest in that investment.
10. Is it possible (given your discussion in the previous problem) that different people might have different required rates of return on the same stock or same series of cash flows?
11. How might your answer to the question above explain how stock prices fluctuate through time?
12. Why doesn't an infinite series of cash flows cost an *infinite* number of dollars today?
13. What must we know about the relationship between a growing annuity's (or a growing perpetuity's) discount rate r and payment growth rate g? What happens if $r = g$?
14. Explain how stocks and bonds can resemble annuities to an investor. How then might we determine what a stock or bond is worth?

ANALYTICAL QUESTIONS

You serve as the agent for a soon-to-be WNBA star. Your client, a generational talent, was drafted first overall and immediately garnered interest from Nike and Adidas. They'd like to sign your client to an endorsement deal (of which you, as the agent, would take a percentage). Your client is indifferent between the two brands and decides that they'd rather be with whichever brand offers more monetary value. Terms of the deals that Nike and Adidas offer are below:

NIKE

Immediately (today), your client will receive a check for $3 million. In one year, your client will receive an end-of-year payment of $250,000. Over the next 4 years, this payment will grow at 5% each year until five total payments have been paid.

ADIDAS

Your client will receive $600,000 annual payments, beginning immediately, collecting a total of 9 equal and annual $600,000 payments.

If your client were to receive all of this money upfront from either Nike or Adidas, she'd be able to invest it at a rate of 5%. Explain to your client:

(1) Which endorsement deal she should select.
(2) Why making the wrong decision could cost her *$287,451!*

NOTES & REFERENCES

[1] See https://www.jgwentworth.com/ and https://www.oasisfinancial.com/.
[2] For the current components of the dividend aristocrats index (which varies through time as firms start and/or stop paying dividends which either qualifies or eliminates them from being a member of the index), see https://www.spglobal.com/spdji/en/indices/dividends-factors/sp-500-dividend-aristocrats/#overview and https://www.spglobal.com/spdji/en/indices/dividends-factors/sp-dividend-monarchs-index/#overview for the dividend monarchs index.