University of Richmond
Robins School of Business™

§7. Compounding Periods and Amortizing Loans

Fin 360: Principles of Financial Management

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INTEREST RATES AND COMPOUNDING PERIODS

So far, we've considered annual time periods and annual interest payments. We can also compute the PV and FV assuming, for example, daily, weekly, monthly, quarterly, semiannual, and even continuous compounding. We need to make simple adjustments to our inputs when considering these other compounding periods.

In practice, rates are often quoted at the **Annual Percentage Rate** (**APR**) or **nominal rate**:

$$APR = Per\ Period\ Interest\ Rate \times Periods\ per\ Year$$

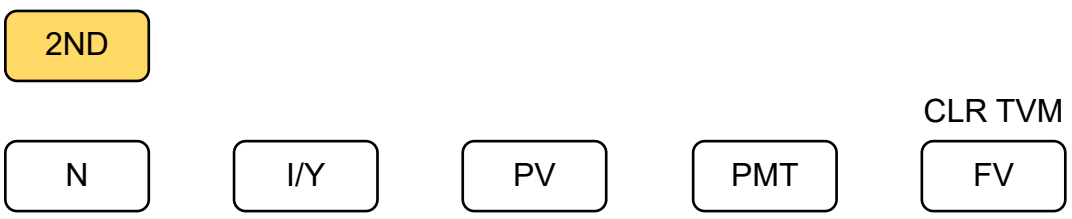
For example, a quarterly 2% interest rate has a 2% × 4 quarters = 8% APR. If given an APR and the number of years, and you'd like to calculate the PV or FV in your calculator with other compounding periods:

Table 1: Compounding Period Conversions

Compounding Period	I/Y = APR divided by...	N = Year multiplied by...
Annually	1	1
Semiannually	2	2
Quarterly	4	4
Monthly	12	12
Weekly	52	52
Daily	365	365

PRACTICE: You deposit $100 in an account paying a 10% APR that compounds daily. How much will you have in the account at the end of 1 year?

SOLUTION: We need to make sure the inputs in the calculator reflect the correct number of compounding periods, and that the interest rate we input is on a per-period basis.



PRACTICE: You are planning on making end-of-period contributions totaling $1,000 each year for 20 years. What would this series of cash flows be worth today if you assume you'd be able to invest them at 4.89% APR, given the following assumptions:

(a) Annual payments and compounding
(b) Semiannual payments and compounding
(c) Quarterly payments and compounding
(d) Monthly payments and compounding
(e) Weekly payments and compounding
(f) Daily payments and compounding

Additionally, determine how much you'd have in each account after all contributions are made.

SOLUTION: We can complete the table below to answer each of these questions (a) through (f), being sure to adjust the number of periods, the interest rate, and payments:

Table 2: Compounding Periods Calculator Inputs Grid

	N	I/Y	PMT	PV	FV
(a)					
(b)					
(c)					
(d)					
(e)					
(f)					

Many online calculators are available to help you compute present and future values of lump sums, annuities, perpetuities, and more. Be cautious with these calculators, as they often differ in how you should properly enter the necessary inputs. Verify their computations with your personal calculator. One full-featured example is at Calculator Soup.[1]

EAR AND APY

The **Effective Annual Rate** (**EAR**) is the *actual* rate one pays or earns considering the compounding periods. The EAR may also be referred to as an **Annual Percentage Yield** (**APY**).

In the example above, with the $100 deposit and 10% annual rate (APR) compounding daily, your "actual" annual rate of return was:

$$\frac{110.5156 - 100}{100} = 10.5156\% = EAR$$

This is your EAR. Your APR as given in the problem was only 10%. Your true rate of return over the course of the year was higher than the APR, given the increased compounding. We can convert from APR to EAR with the following formulas, *ensuring that we first convert our percentages into decimals before using the formulas*:

$$EAR = \left[1 + \frac{APR}{m}\right]^m - 1 \qquad \leftrightarrow \qquad APR = \left(\sqrt[m]{EAR + 1} - 1\right) \times m$$

where *m* is the number of compounding periods. For *continuous* compounding we make use of the **exponential function *e*** ≈ 2.718281828 (available on your calculator.)

$$EAR = e^{APR} - 1 \qquad \leftrightarrow \qquad APR = \ln\left(EAR + 1\right)$$

PRACTICE: Below is a table of APR and EAR conversions. Using the conversion formulas, determine the values that belong in the missing cells. On your own, verify the other cells of the table.

Table 3: EAR and APR Conversions

		EAR			
		Daily	**Monthly**	**Quarterly**	**Continuously**
APR	7%		7.2290%	7.1859%	7.2508%
	8%	8.3278%	8.3000%	8.2432%	
	9%	9.4162%	9.3807%	9.3083%	9.4174%

SOLUTION: To illustrate using 7% APR with daily compounding:

$EAR =$

$APR =$

To illustrate using 8% APR with continuous compounding:

$EAR =$

$APR =$

Use EARs to compare options. Would you prefer (1) a savings account offering a 16% APR that compounds annually, or (2) a savings account with a 15% APR that compounds daily?

(1) has an EAR of 16% while (2) has an EAR of 16.18%. Therefore, you should choose (2) even though it has a *lower* APR because of the compounding!

APPLICATION: CAR LOANS

PRACTICE: Calculate your monthly payments and EAR on a new Tesla Cybertruck. The truck costs $99,990, plus an additional destination fee of $1,995. You will be expected to pay a down payment of $3,999 at the purchase. The APR is 5.99%, and the loan term is 72 months.

SOLUTION: First, we determine that the loan amount is _____, or the cost plus the destination fee and minus the down payment. Then,

2ND

CLR TVM

N | I/Y | PV | PMT | FV

For the effective annual rate,

$$EAR = \left[1 + \frac{APR}{m}\right]^{m} - 1 =$$

INTERPRETATION: Note that the inputs for the calculator must be in the same units. The N, I/Y, and PMT are all per month. The loan amount is input as a positive cash flow because we receive that amount today for the car, with the payments as "negative" to represent cash outflows paid. Although we are quoted a 5.99% annual rate, we are effectively paying a rate of EAR = _____% per year given we are carrying a balance on the loan from month to month and pay interest monthly.

How is the EAR the "actual rate you pay"? Consider an example: a one-year $100 loan charges 5% APR with quarterly payments and compounding:

N = 4
I/Y = 1.25 (or 5% ÷ 4)
PMT = -1.25 (or $100 × 0.05 ÷ 4), only the interest portion
FV= <CPT> 5.0945

The future value of the $1.25 quarterly payments is $5.0945, or 5.0945% of the $100 loan in the first year.

Similarly, the EAR = $\left[1 + \frac{0.05}{4}\right]^{4} - 1 = 5.0945\%$

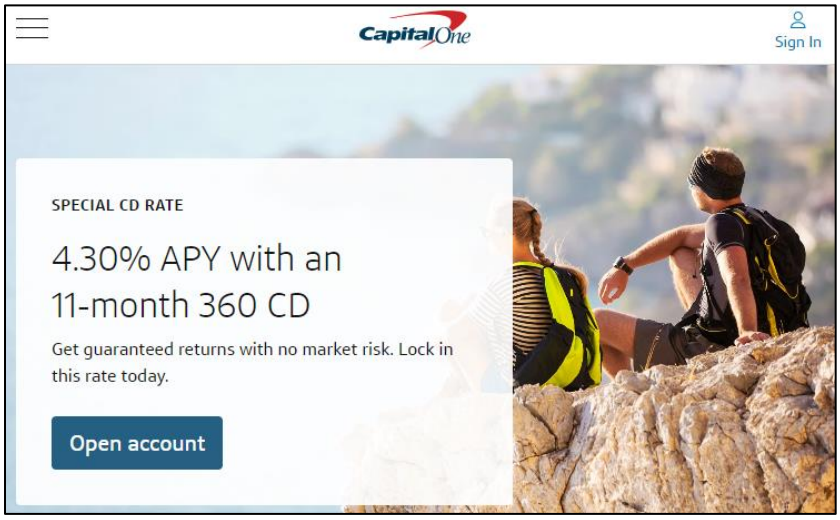
You interest payments sum to 5.0945% of the loan, not just 5%.

TRUTH IN LENDING ACT

Surprisingly, the Federal **Truth-in-Lending Act** requires that lenders disclose APR[2], *not* EARs, unambiguously and prominently on loan documents. Yet, as we have seen, the effective rate that you pay may be higher than the APR lenders are required to disclose.

By these same laws, banks present the EAR (which they often call an **APY**) at which you can save by depositing to their accounts. This represents the actual interest rate you will earn over the period. **Certificates of Deposit** are time deposits at a bank, paying slightly higher than savings accounts because the bank "locks up" your money for a specified period. Withdrawing early requires you to pay penalties.

Figure 1: Certificate of Deposit Advertisement



Why are banks willing to pay more on CDs? Why would you require a CD have a higher interest rate than a savings account?

EXAMPLE: A payday lending company lends you $100 today if you agree to pay them back $115 in two weeks after you receive your next paycheck. What's the APR on this loan? What's the EAR?

SOLUTION: We can find the APR and EAR by the formulas we use above:

$$APR = 15\% \times 26\ biweekly\ periods\ in\ a\ year = 390\%$$

$$EAR = \left[1 + \frac{3.90}{26}\right]^{26} - 1 \ = 36.8568 = 3{,}685.68\%$$

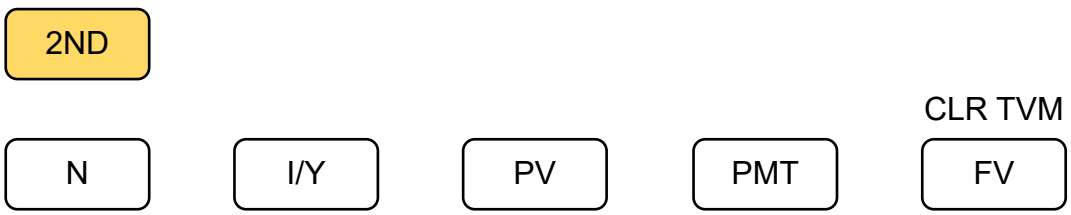
INTERPRETATION: While paying $15 on a $100 loan may seem reasonable, given that 15% is over the course of only 2 weeks, the actual annualized interest rate is substantial. Payday loan limits vary by state, and may have maximums of $10 to $30[3] of interest over a two-week period for every $100 borrowed.

AMORTIZING LOANS

Amortizing loans are loans that pay principal down through time. Each period, interest is calculated on the remaining principal on the loan. Common amortizing loans include most standard mortgages and car loans.

PRACTICE: You wish to purchase a home for $750,000. Truist Bank currently offers a 30-year Fixed Mortgage (i.e., a fixed interest rate and fixed payment each month) with an APR of 5%. How much will your monthly payment be? How much do you pay in total for this $750,000 property? Assume your down payment is 20% of the home price.

SOLUTION: We can calculate the payments in the financial calculator:



Next, we can complete an **Amortization Schedule** which shows how our fixed payment consists of a principal and interest component in each month

Table 4: Amortization Schedule

Month	Beginning Balance	Total Payment	Interest Paid	Principal Payment	Ending Balance
1	$600,000	= [+]			
2					
⋮	⋮	⋮	⋮	⋮	⋮
360	$3,207.56		$13.36	$3,207.56	$0

Our finanical calculator can help us fill out any row of an amortization schedule using the **AMORT** function above the PV key. First solve for the payment as we've done above. Then, press **2ND** → **AMORT**.

Suppose we wish to fill out the row below for the 50th month. We type **50** **ENTER** for **P1**, then ↓ **50** **ENTER** for **P2**, indicating we want to view the totals for the 50th period. Pressing ↑ ↓ allows us to cycle through the ending balance, principal payment, and interest payment for the 50th period. The total payment should be the same as above, and the beginning balance for the 50th period is the same as the ending balance for the 49th period.

Month	Beginning Balance	Total Payment	Interest Paid	Principal Payment	Ending Balance
50					

To obtain the total interest and principal paid over the life of the 360 months in this loan, we choose 1 for **P1** and 360 for **P2**.

Total Interest Paid = _____ Total Principal Paid = _____

Total Paid Overall = _____

INTERPRETATION: While the payment overall remains constant, a majority of the payment applies to interest at the beginning of the loan term while a majority of the payment applies to the principal near the end of the loan term. Notice how substantial the amount of interest paid over the life of the loan is relative to the principal amount. Total interest paid can even *exceed* that of the principal paid over the life of the loan.

Verify these computations and your amortization schedule in an online calculator, such as the one available at Calculator.net.[4]

PRACTICE: Would you prefer the loan above ($600k for 30 years, paid monthly, at 5% APR) or a loan for the same amount over 15 years paid monthly at a 7% interest rate?

SOLUTION: We know the payment for the loan in the previous example is $3,220.93. By the same method, we determine that the payments for the loan in the case of $600,000 at 7% APR for 15 years is:

Payments for 7% Loan = _____

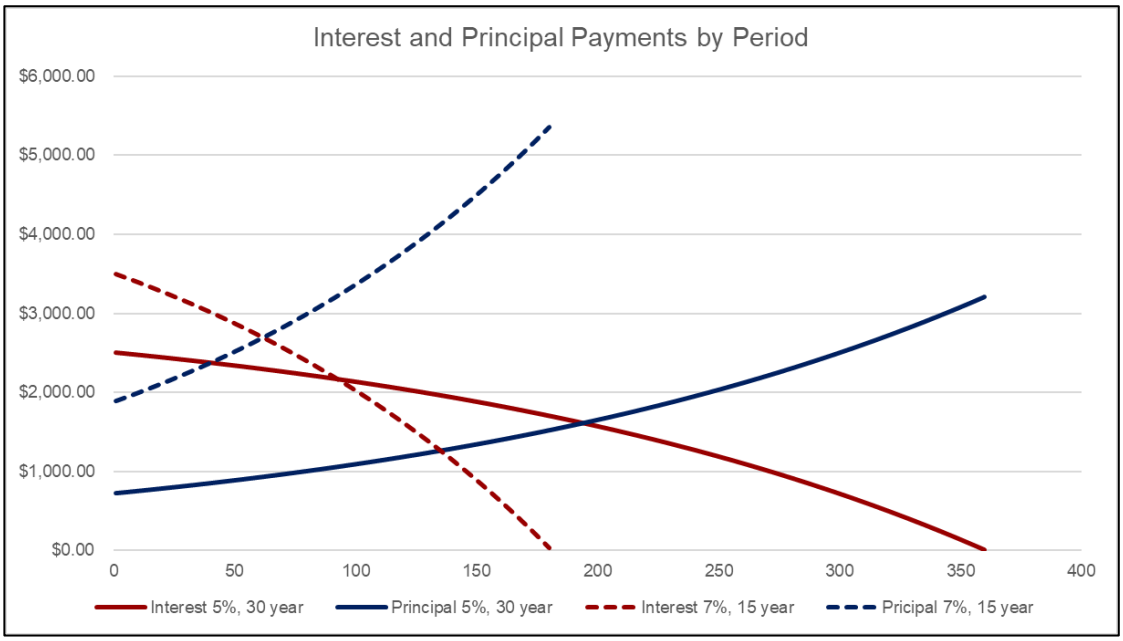
We consider the total amount of the interest paid over the life of this loan by setting **P1** and **P2** as the beginning and ending periods.

Total Interest Paid = _____ Total Principal Paid = _____

Total Paid Overall = _____

INTERPRETATION: Which would you prefer? The shorter term loan indeed has a lower amount of interest paid over time despite the higher rate, but based on what you are able to afford each month, you may choose the lower overall payment (at *significant* long-term expense!)

Figure 2: Amortizing Loans Comparison



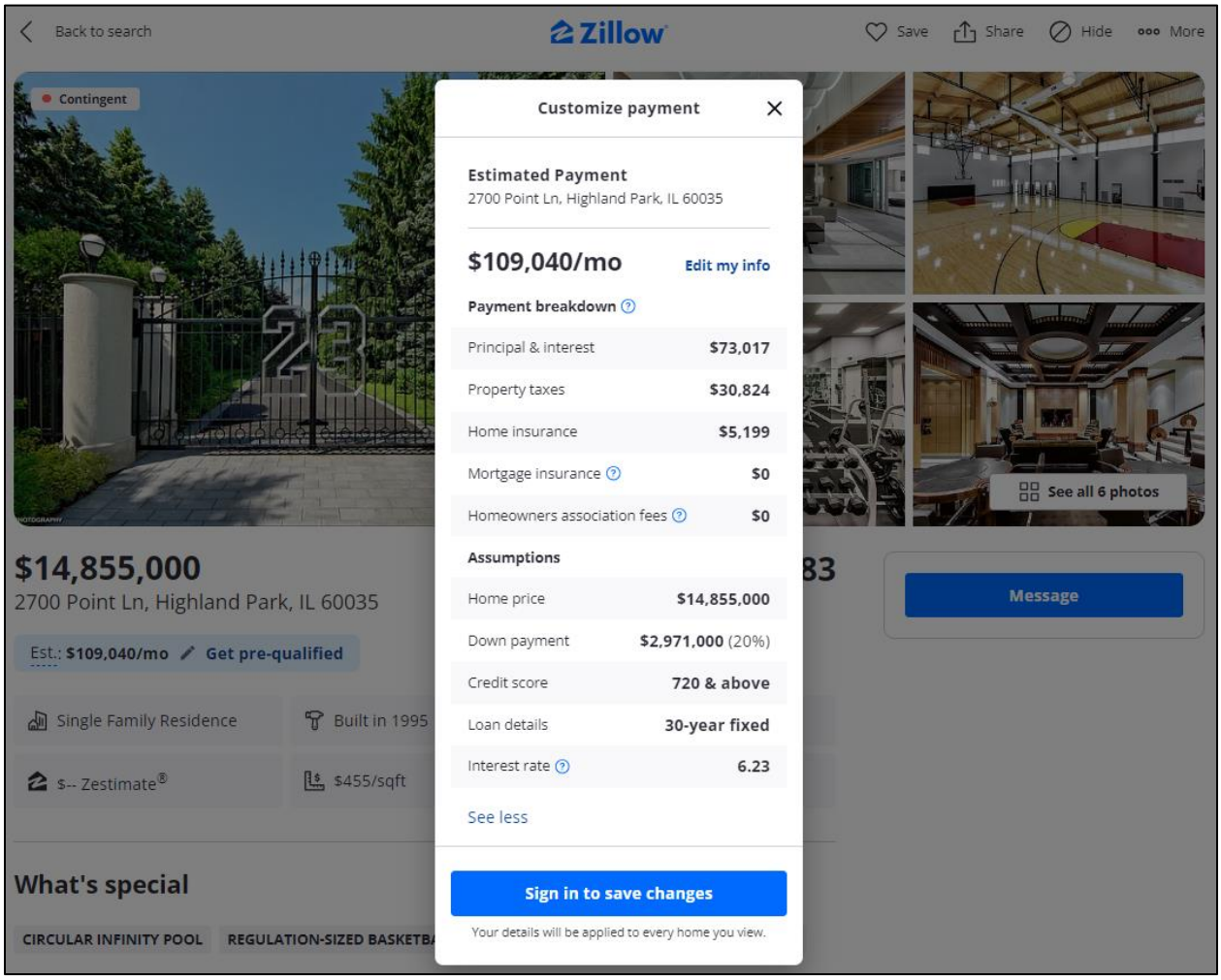
For additional practice where you can input your own amortizing loan terms, see the Excel file *Amortizing Loan* at www.josephfarizo.com/fin360.html.

CRITICAL THINKING & CONCEPTUAL QUESTIONS

1. Why should the EAR be used to compare loans or bank accounts instead of APR?
2. Can the EAR ever be less than the APR? Can the EAR and the APR ever be the same?
3. Why do lenders present the APR and savings institutions present the EAR? Why might this be problematic for consumers?
4. What effect does increasing compounding periods have on present values and future values?
5. Explain the concept of *continuous* compounding. Does it ever happen in practice?
6. What is an APY?
7. What are the pros and cons of a certificate of deposit relative to saving in a traditional bank account?
8. Why would a bank offer a CD? What is the advantage from their perspective?
9. Why would a bank offer continuous compounding to a saver if they know that annual or monthly compounding would result in their paying less interest?
10. If you decide to pay several months of your mortgage early, why is it important that these payments be applied to the mortgage's *principal* and not the mortgage's interest?
11. In an amortizing loan, why is a greater portion of the monthly payment applied to interest than the principal? Through time, why is a greater portion of the monthly payment applied to the principal than the interest?
12. Why is the interest paid over the life of an amortizing loan not just the interest rate multiplied by the principal? For example, one would pay \$529,200 in interest on a 30-year \$400,000 mortgage with a 6.70% APR, not simply \$400,000 × 0.0670 = \$26,800 in interest. (*Yes*, you pay \$529,200.29 + \$400,000 = \$929,200.29 for a \$400,000 house!)

ANALYTICAL QUESTIONS

Michael Jordan's Highland Park home has been on the market for years. For the low price of $109,040/month, you could be the new owner! Calculate for yourself that the monthly mortgage payment would be $73,017 from the screenshot below, given the assumptions that Zillow provides. What other significant expenses would increase the value of the monthly payments (to get to a total of over $109,040/month)? How much would you pay in interest alone over the life of this mortgage?



NOTES & REFERENCES

[1] Calculator Soup: https://www.calculatorsoup.com/calculators/financial/index-time-value-of-money-calculators.php
[2] What is a Truth-in-Lending Disclosure? When Do I Get to See It? *Consumer Financial Protection Bureau*: https://www.consumerfinance.gov/ask-cfpb/what-is-a-truth-in-lending-disclosure-when-do-i-get-to-see-it-en-787/
[3] What is a Payday Loan? *Consumer Financial Protection Bureau*: https://www.consumerfinance.gov/ask-cfpb/what-is-a-payday-loan-en-1567/#:~:text=Many%20state%20laws%20set%20a,percent%20to%20about%2030%20percent.
[4] Amortization calculator: https://www.calculator.net/amortization-calculator.html.