UNIVERSITY OF RICHMOND
Robins School of Business™

BOND VALUATION

FIN 360: PRINCIPLES OF FINANCIAL MANAGEMENT

BOND VALUATION AND YIELD TO MATURITY

PRACTICE: Microsoft sells a 10-year bond with a 3.3% coupon (or 3.3% ÷ 2 = 1.65% every 6 months) for "general corporate purposes" including the repayment of short-term debt and paying for the acquisition of LinkedIn.

You pay the bond's **par value** of $1,000. What is the bond's **yield to maturity** – or its annual rate of return? What if you instead pay $900 on the **secondary market** immediately after the bond is issued? What if you pay $1,100?

SOLUTION: As we've seen, the timeline is:

Year	0	0.5	1	1.5	2	…	8	8.5	9	9.5	10
Coupon		$16.50	$16.50	$16.50	$16.50		$16.50	$16.50	$16.50	$16.50	$16.50
Face Value											$1,000
Cash Flow	*($1,000)*	*$16.50*	*$16.50*	*$16.50*	*$16.50*		*$16.50*	*$16.50*	*$16.50*	*$16.50*	*$1,016.50*

A bond's value is determined as the

1. Present Value of the Coupons
2. Present Value of the Principal

$$Bond\ Value = \left[C \times \frac{1 - \frac{1}{(1+r)^t}}{r} \right] + \frac{FV}{(1+r)^t}$$

Therefore,

$$\$1{,}000 = \left[16.50 \times \frac{1 - \frac{1}{(1+r)^{20}}}{r}\right] + \frac{1{,}000}{(1+r)^{20}}$$

Notice the payments are *semiannual* and the number of periods is 20 *semiannual* periods across 10 years. If we solve for r,

$$r = 0.0165 = 1.65\%$$

The yield to maturity is always expressed as an annual rate, but this was computed based on semiannual payments. We multiply by 2 to express as an annual rate:

$$YTM = 0.0165 \times 2 = 3.3\%$$

If you pay \$900:

$$\mathbf{\$900} = \left[16.50 \times \frac{1 - \frac{1}{(1+r)^{20}}}{r}\right] + \frac{1{,}000}{(1+r)^{20}}$$

$$r = 0.02278 = 2.278\%$$

$$YTM = 0.02278 \times 2 = 4.556\%$$

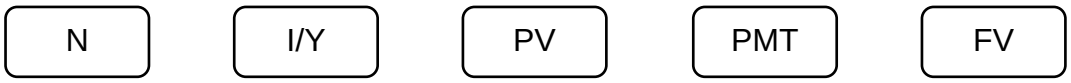
If you pay \$1,100:

$$\mathbf{\$1,100} = \left[16.50 \times \frac{1 - \frac{1}{(1+r)^{20}}}{r}\right] + \frac{1{,}000}{(1+r)^{20}}$$

$$r = 0.01091 = 1.091\%$$

$$YTM = 0.01091 \times 2 = 2.182\%$$

We'll use our calculators to solve for this YTM, recognizing that the PMT must be expressed semiannually, and the resulting I/Y converted to an annual rate:



INTERPRETATION:

To summarize,

Bond Price	YTM	Explanation
$900	4.556%	Purchased **at a discount**: YTM > Coupon Rate.
$1,000	3.3%	Purchased **at par**: YTM = Coupon Rate.
$1,100	2.182%	Purchased **at a premium**: YTM < Coupon Rate.

As you pay more for the bond, the yield to maturity (rate of return) falls. As you pay less for a bond, the yield to maturity (rate of return) rises.

Bond prices and yields are inversely related. The more you pay, the lower your yield.

Remember that (a) coupons are quoted as *annual* rates and need to be converted to semiannual payments by dividing by 2. Then, (b) the I/Y must be converted back to an *annual* rate by multiplying by 2 to get the YTM.

PRACTICE: You purchase an Apple bond on the secondary market that has 6 years left to maturity and pays 8% coupons on a face value of $5,000. You pay a price such that the bond is yielding 8.74%. Will this be a **premium** or **discount** bond? What did you pay for this bond?

SOLUTION: The coupon is _____ than the yield. This must be a _____ bond.

To find the price:

N	I/Y	PV	PMT	FV

CURRENT YIELD

A bond's **current yield**, not to be confused with the **yield to maturity** or just "**yield**", is the bond's *annual coupon* divided by the bond's current price. For the Apple bond above:

$$Current\ Yield = \frac{Annual\ Coupon}{Price} = \frac{\qquad\qquad}{\qquad\qquad} =$$

This tells us the annual coupon payment as a percentage of the price you pay.

Determining the Discount Rate

Practice: How much should you pay for a 30-year, 5% coupon Microsoft bond with a $1,000 face value that was issued 5 years ago and has 25 years left to maturity? Apple just issued a similar bond with a 7% coupon.

Solution: If Apple, who we assume is a *reasonably similar* company to Microsoft, just issued a *reasonably similar* bond in the current interest rate environment that pays 7%, we should apply that discount rate to the Microsoft bond.

N	I/Y	PV	PMT	FV

Interpretation: Companies issue bonds at *prevailing market rates*. That is, if interest rates in the economy are such that investors expect a 7% return from investment grade debt, then investment grade companies will issue their new bonds with a coupon around 7%. Through time, interest rates change, but *previously* issued bonds' coupons remain the same. Therefore, the prices that investors are willing to pay for the bond in the secondary market will fluctuate such that the yield is similar to what the prevailing interest rate is in the economy.

☆ Bonds of similar risk and structure (maturity, callability, etc.) will have prices in markets such that they yield about the same return, regardless of the coupon rate.

We can make the following claims about the relationships of bond prices and their yields:

Premium Bonds: $Coupon\ Rate > Current\ Yield > YTM\ and\ Price > Par$

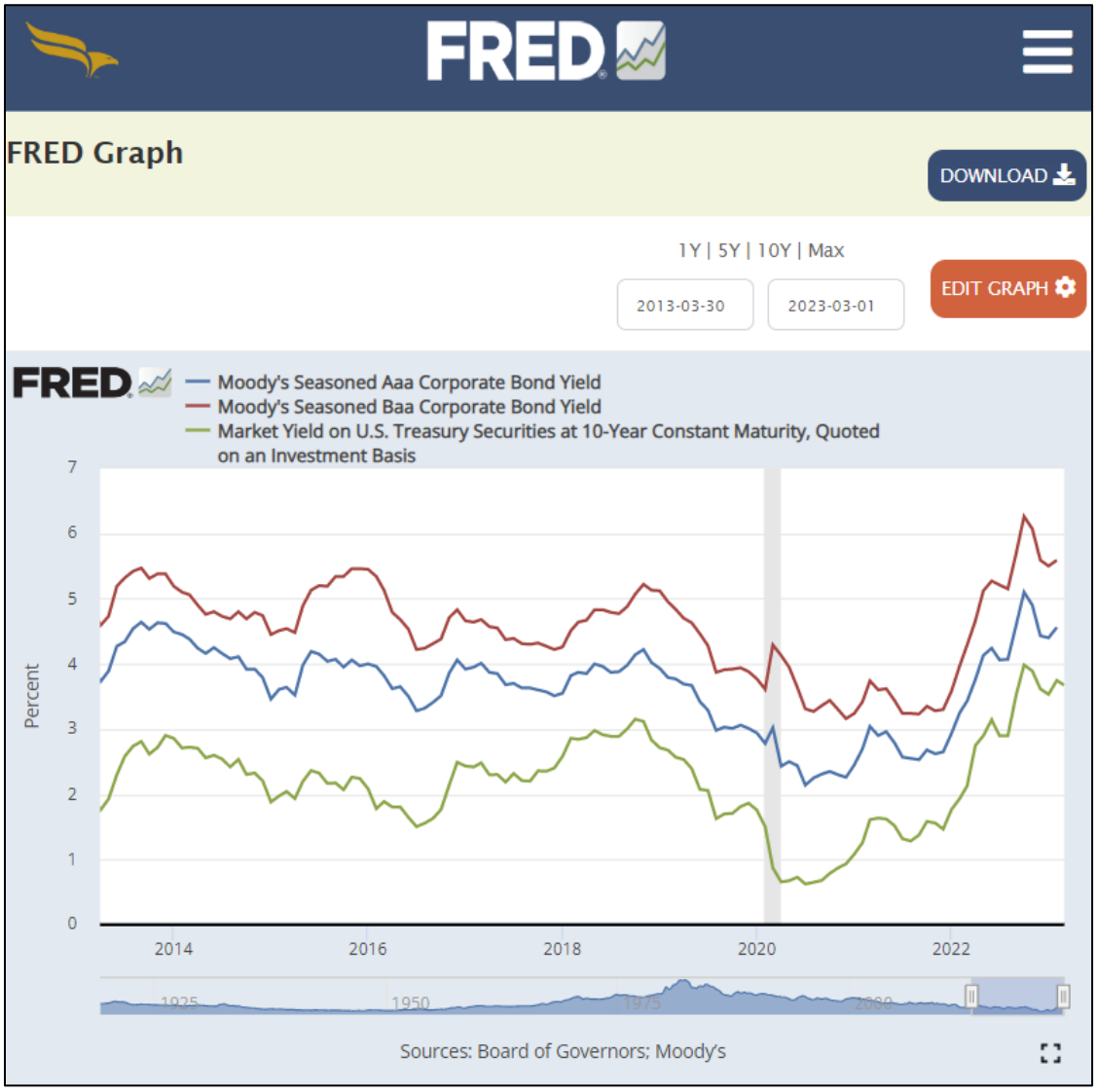
and

Discount Bonds: $Coupon\ Rate < Current\ Yield < YTM\ and\ Price < Par$

For a dynamic bond price and yield calculator, see the Excel file *Bond Price and Yield Calculator* at www.josephfarizo.com/fin360.html.

We can obtain current and historical bond yields for corporate and government issuer on the **St. Louis FED's FRED** website: https://fred.stlouisfed.org/. The existing rates give us estimates of appropriate discount rates to use in our bond valuations.

Figure 1: Bond Yields



FACTORS INFLUENCING BOND YIELDS

There are four key risks influencing bond yields:

1. **Interest Rate Risk**: the risk that rising interest rates will reduce the value of the bonds you hold. For example, if you bought a 30-year $1,000 bond with 5% coupons and interest rates rise in the economy to 7%, your bond becomes less valuable than the new bonds issued with coupons nearer to 7%. If you wanted to sell your bond, you'd have to lower its price such that the buyer's yield would be close to 7%.

2. **Credit or Default Risk**: the risk that the bond issuer will be unable to pay the coupons or principal.

3. **Liquidity Risk**: the risk that your bond can't be sold quickly without a significant loss in value.

4. **Inflation Risk**: the risk that rising prices will erode the purchasing power of your bond's coupon payments.

Longer term bonds are generally more susceptible to each type of risk. Their prices swing more when rates change than shorter term bonds do.

INFLATION AND THE FISHER EFFECT

Nominal rates are the quoted rates of interest and return that provide actual returns in dollars, while **real rates** are the returns in terms of purchasing power, inclusive of the effects of inflation.

The **Fisher Effect**, named after the economist Irving Fisher, shows the relationship between nominal rates R, real rates r, and the inflation rate h as:

$$1 + R = (1 + r) \times (1 + h)$$

If, for example, a bond sells as par and pays a 5% coupon, your nominal rate is 5%. If you expect inflation to be 3%, your real rate of return will be:

$$1 + R = (1 + r) \times (1 + h)$$

$$1 + 0.05 = (1 + r) \times (1 + 0.03)$$

$$1.05 = (1 + r) \times (1.03)$$

$$\frac{1.05}{1.03} = (1 + r)$$

$$\frac{1.05}{1.03} - 1 = r$$

$$r = 0.01942$$

Or, your purchasing power is expected to increase by 1.942% even though the bond returns 5%. In practice, we should be careful to consider the effects of inflation, as nominal returns alone cannot guarantee an increase in (or even preservation of) purchasing power.