University of Richmond
Robins School of Business™

Compounding Periods and Amortizing Loans

Fin 360: Principles of Financial Management

INTEREST RATES AND COMPOUNDING PERIODS

So far, we've generally considered *annual* time periods and *annually*-made interest payments. We can also compute the PV and FV assuming, for example, daily, weekly, monthly, quarterly, semiannual, and even *continuous* compounding. We need to make simple adjustments to our inputs when considering these other compounding periods.

In practice, rates are often quoted at the **Annual Percentage Rate** (**APR**) or **nominal rate**:

$$APR = Per\ Period\ Interest\ Rate \times Periods\ per\ Year$$

For example, a quarterly 2% interest rate has a 2% × 4 quarters = 8% APR. If given an APR and the number of years, and you'd like to calculate the PV or FV in your calculator with other compounding periods:

Compounding Period	I/Y = APR divided by…	N = Year multiplied by…
Annually	1	1
Semiannually	2	2
Quarterly	4	4
Monthly	12	12
Weekly	52	52
Daily	365	365

PRACTICE: You deposit $100 in a bank paying a 10% annual rate that *compounds daily*. How much will you have in the account at the end of 1 year?

N | I/Y | PV | PMT | FV

PRACTICE: You are planning on making end-of-period contributions *totaling* $1,000 each year for 20 years. What would this series of cash flows be worth *today* if you assume you'd be able to invest them at 4.89% per year, given the following assumptions:

(a) Annual payments and compounding
(b) Semiannual payments and compounding
(c) Quarterly payments and compounding
(d) Monthly payments and compounding
(e) Weekly payments and compounding
(f) Daily payments and compounding

Additionally, determine how much you'd have in each account after all contributions are made.

SOLUTION: Be sure to adjust the number of periods, the interest rate, and payments:

	N	I/Y	PMT	PV	FV
(a)					
(b)					
(c)					
(d)					
(e)					
(f)					

Numerous online calculators are available to help you compute present and future values of lump sums, annuities, perpetuities, and more. Be cautious with these calculators, as they often differ in how one should properly enter the necessary inputs. Verify their computations with your personal calculator. One full-featured example: https://www.calculatorsoup.com/calculators/financial/index-time-value-of-money-calculators.php

APR, EAR, AND APY

The **Effective Annual Rate** (**EAR**) is the *actual* rate one pays or earns considering the compounding periods. The EAR may also be referred to as an **Annual Percentage Yield** (**APY**).

In the example above, with the $100 deposit and 10% annual rate (APR) compounding daily, your "actual" annual rate of return was:

$$\frac{110.5156 - 100}{100} = 10.5156\% = EAR$$

This is your EAR. Your APR as given in the problem was only 10%. Your true rate of return over the course of the year was higher than the APR, given the increased compounding. We can convert from APR to EAR with the following formulas, *ensuring that we first convert our percentages into decimals before using the formulas*:

$$EAR = \left[1 + \frac{APR}{m}\right]^m - 1 \qquad \leftrightarrow \qquad APR = \left(\sqrt[m]{EAR + 1} - 1\right) \times m$$

where *m* is the number of compounding periods. For *continuous* compounding we make use of the **exponential function *e*** ≈ 2.718281828 (available on your calculator.)

$$EAR = e^{APR} - 1 \qquad \leftrightarrow \qquad APR = \ln(EAR + 1)$$

EXAMPLE: Below is a table of APR and EAR conversions based on the formulas above. On your own, verify each cell of the table.

		EAR			
		Daily	**Monthly**	**Quarterly**	**Continuously**
APR	7%	7.2501%	7.2290%	7.1859%	7.2508%
	8%	8.3278%	8.3000%	8.2432%	8.3287%
	9%	9.4162%	9.3807%	9.3083%	9.4174%

To illustrate using the 7% APR with daily compounding:

$$EAR = 7.2501\% = \left[1 + \frac{0.07}{365}\right]^{365} - 1 \quad \leftrightarrow \quad APR = 7\% = 365 \times \sqrt[365]{0.072501 + 1} - 1$$

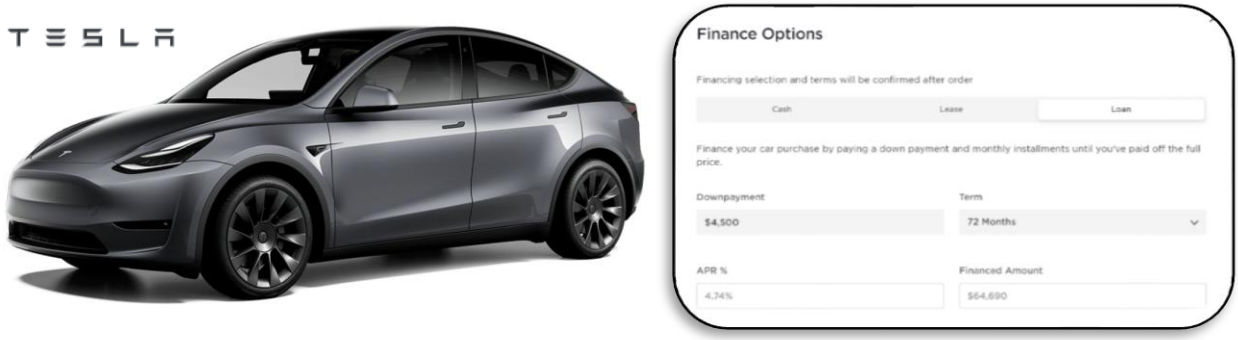
To illustrate using the 7% APR with continuous compounding:

$$EAR = 7.25082\% = e^{0.07} - 1 \quad \leftrightarrow \quad APR = 7\% = \ln(0.072508 + 1)$$

Would you prefer a savings account offering a 16% APR that compounds annually or a savings account offering a 15% APR that compounds daily? Compute their EARs to compare.

The 16% APR with annual compounding has an EAR of 16% while the 15% APR with daily compounding has an EAR of 16.18%. Choose the *lower* APR option in this case, because its *effective* rate is higher!

PRACTICE: Calculate your monthly payments on a new Tesla Model Y. You've chosen the Long Range Model that starts at $69,190 with a required down payment of $4,500. The loan term is 6 years with a 4.74% APR. What is your *effective* annual rate?



SOLUTION: First, we determine that the loan amount is _____, or the total cost minus what we pay in cash upfront. Then,

N	I/Y	PV	PMT	FV

For the effective annual rate,

$$EAR = \left[1 + \frac{APR}{m}\right]^{m} - 1 = \qquad\qquad =$$

INTERPRETATION: Note that the inputs for the calculator must be in the same *units*. The N, I/Y, and PMT are all *per month*. The loan amount is input as a positive cash flow because we receive that amount today for the car, with the payments as "negative" to represent cash outflows paid. Although we are quoted a 4.74% annual rate, we are *effectively* paying a rate of EAR = 4.84434% per year given we are carrying a balance on the loan from month to month and pay interest monthly.

How is the EAR the "actual rate you pay"? Consider an example. A one-year $100 loan charges 5% APR with quarterly payments and compounding:

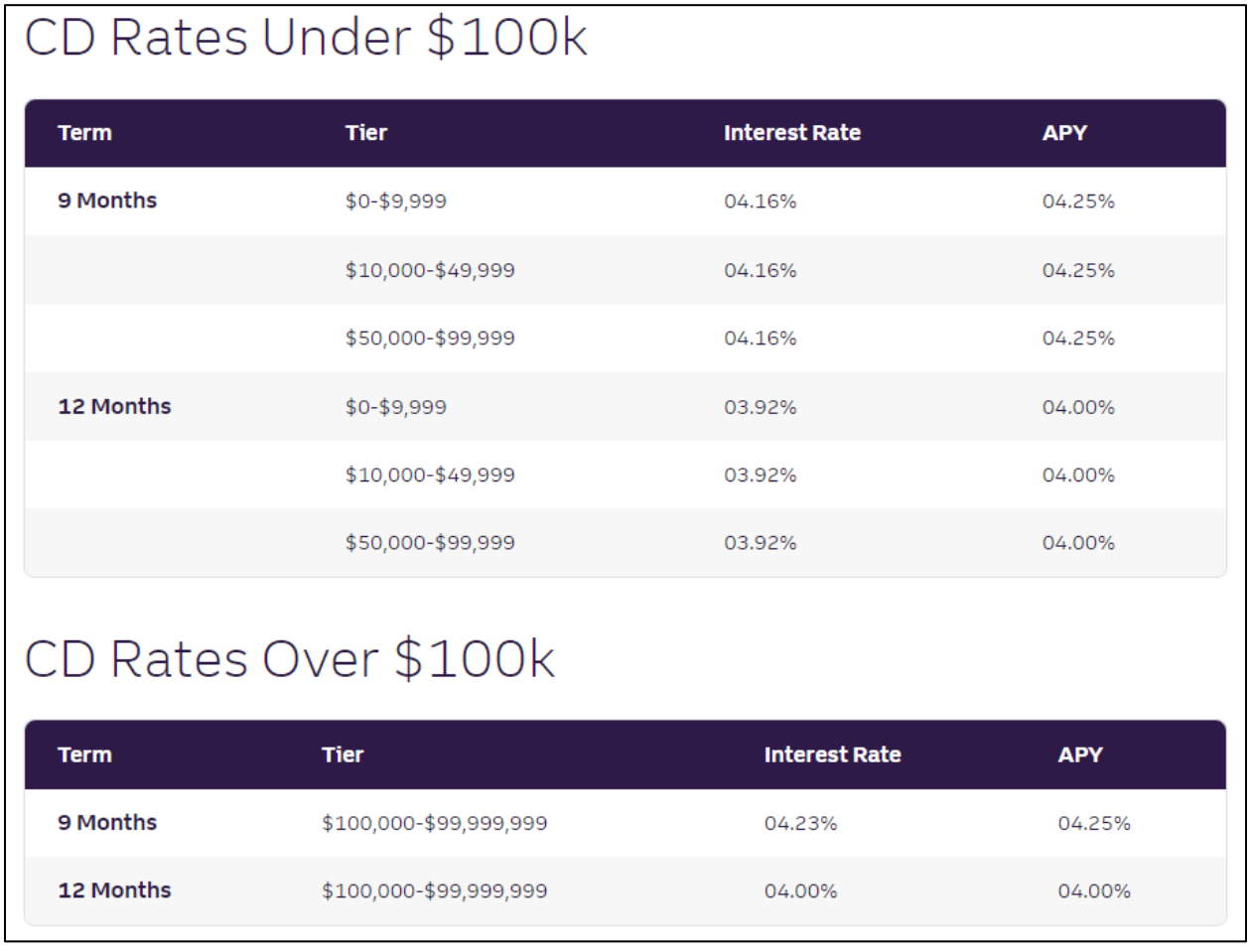
N	I/Y	PV	PMT	FV
4	1.25		-$1.25	<CPT> $5.0945

The future value of the $1.25 quarterly payments is $5.0945, or 5.0945% of the $100 loan in the first year. Similarly, the EAR = $\left[1 + \frac{0.05}{4}\right]^{4} - 1 = 5.0945\%$. Your interest payments, "sum to" 5.0945% of the principal, *not* just 5%.

Surprisingly, the Federal **Truth-in-Lending Act** requires that lenders disclose APR[1], *not* EARs, unambiguously and prominently on loan documents. Yet, as we have seen, the effective rate that you actually pay may be higher than the APR lenders are required to disclose.

By these same laws, banks present the EAR (which they often call an **APY**) at which you can save by depositing to their accounts. This represents the actual interest rate you will earn over the period. **Certificates of Deposit** are time deposits at a bank, paying slightly higher than savings accounts because the bank "locks up" your money for a specified period of time that you choose.

Figure 1: Certificate of Deposit (CD) Rates



CD Rates Under $100k

Term	Tier	Interest Rate	APY
9 Months	$0-$9,999	04.16%	04.25%
	$10,000-$49,999	04.16%	04.25%
	$50,000-$99,999	04.16%	04.25%
12 Months	$0-$9,999	03.92%	04.00%
	$10,000-$49,999	03.92%	04.00%
	$50,000-$99,999	03.92%	04.00%

CD Rates Over $100k

Term	Tier	Interest Rate	APY
9 Months	$100,000-$99,999,999	04.23%	04.25%
12 Months	$100,000-$99,999,999	04.00%	04.00%

PRACTICE: A payday lending company lends you $100 today for the promise that you will pay them back $115 in two weeks after your next paycheck. What's the APR? What's the EAR?

$$APR = 15\% \times 26\ biweekly\ periods\ in\ a\ year = 390\%$$

$$EAR = \left[1 + \frac{3.90}{26}\right]^{26} - 1 \ = 36.8568 = 3{,}685.68\%$$

INTERPRETATION: While paying $15 on a $100 loan may seem reasonable, given that 15% is over the course of only 2 weeks, the actual annualized interest rate is substantial. Payday loan limits vary by state, and may have maximums of $10 to $30[2] of interest over a two-week period for every $100 borrowed.

AMORTIZING LOANS

Amortizing loans are loans that generally pay principal down through time. Each period, interest is calculated on the remaining principal on the loan. Popular amortizing loans include most standard mortgages and car loans.

PRACTICE: You wish to purchase a home for $250,000. Truist Bank currently offers a 30-year Fixed Mortgage (i.e., a fixed interest rate and fixed payment each month) with an APR of 5%. How much will your monthly payment be? How much do you pay in total for this $250,000 property? Assume your down payment is 20% of the home price.

SOLUTION: We can calculate our payments in the financial calculator:

N	I/Y	PV	PMT	FV

Next, we can complete an **Amortization Schedule** which shows how our fixed payment consists of a principal and interest component in each month.

Month	Beginning Balance	Total Payment	Interest Paid	Principal Payment	Ending Balance
1	$200,000	= [+]			
2					
⋮	⋮	⋮	⋮	⋮	⋮
360	$1,069.19		$4.45	$1069.19	$0

Verify these computations and your schedule in an online calculator such as the one available at https://www.calculator.net/amortization-calculator.html.

Our finanical calculator can help us fill out any row of an amortization schedule using the **AMORT** function above the PV key. First solve for the payment as we've done above. Then, press **2ND** → **AMORT**.

Suppose we wish to fill out the row below for the 50th month. We type **50** **ENTER** for **P1**, then ↓ **50** **ENTER** for **P2**, indicating we want to view the totals for the 50th period. Pressing ↑ ↓ allows us to cycle through the ending balance, principal payment, and interest payment for the 50th period. The total payment should be the same as above, and the beginning balance for the 50th period is the same as the ending balance for the 49th period.

Month	Beginning Balance	Total Payment	Interest Paid	Principal Payment	Ending Balance
50					

To obtain the total interest and principal paid over the life of the 360 months in this loan, we choose 1 for **P1** and 360 for **P2**.

Total Interest Paid = _____ Total Principal Paid = _____

Total Paid Overall = _____

For additional practice where you can input your own terms, see the Excel file *Amortizing Loan* at www.josephfarizo.com/fin360.html

PRACTICE: Would you prefer the loan above ($200k for 30 years, paid monthly, at 5% APR) or a loan for the same amount over 15 years paid monthly at a 7% interest rate?

SOLUTION: We know the payment for the loan in the previous example is $1,073.64. By the same method, we determine that the payments for the loan in the case of $200,000 at 7% APR for 15 years is:

_____

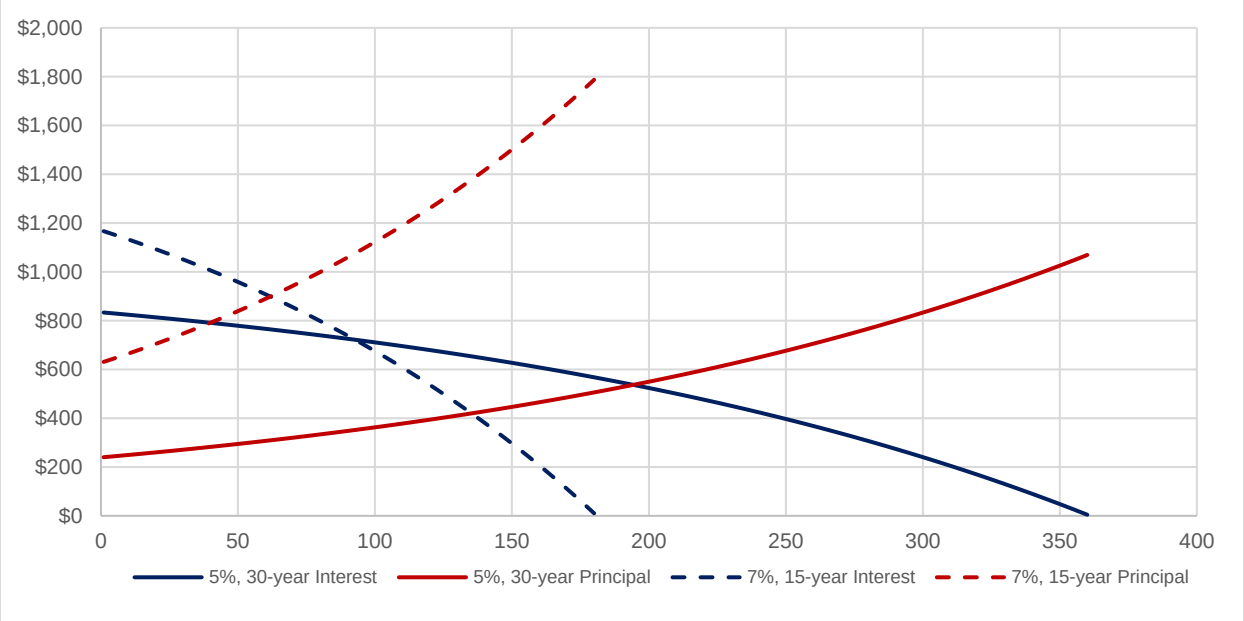
We consider the total amount of the interest paid over the life of this loan by setting **P1** and **P2** as the beginning and ending periods.

Total Interest Paid = _____ Total Principal Paid = _____

Total Paid Overall = _____

INTERPRETATION: Which would you prefer? The shorter term loan indeed has a lower amount of interest paid over time despite the higher rate, but based on what you are able to afford each month, you may choose the lower overall payment (at *significant* long-term expense!)

Figure 2: Amortization Figure



REFERENCES

[1] What is a Truth-in-Lending Disclosure? When Do I Get to See It? *Consumer Financial Protection Bureau*: https://www.consumerfinance.gov/ask-cfpb/what-is-a-truth-in-lending-disclosure-when-do-i-get-to-see-it-en-787/
[2] What is a Payday Loan? *Consumer Financial Protection Bureau*: https://www.consumerfinance.gov/ask-cfpb/what-is-a-payday-loan-en-1567/#:~:text=Many%20state%20laws%20set%20a,percent%20to%20about%2030%20percent.