University of Richmond
Robins School of Business™

Discounted Cash Flow Valuation

Fin 360: Principles of Financial Management

MULTIPLE CASH FLOW

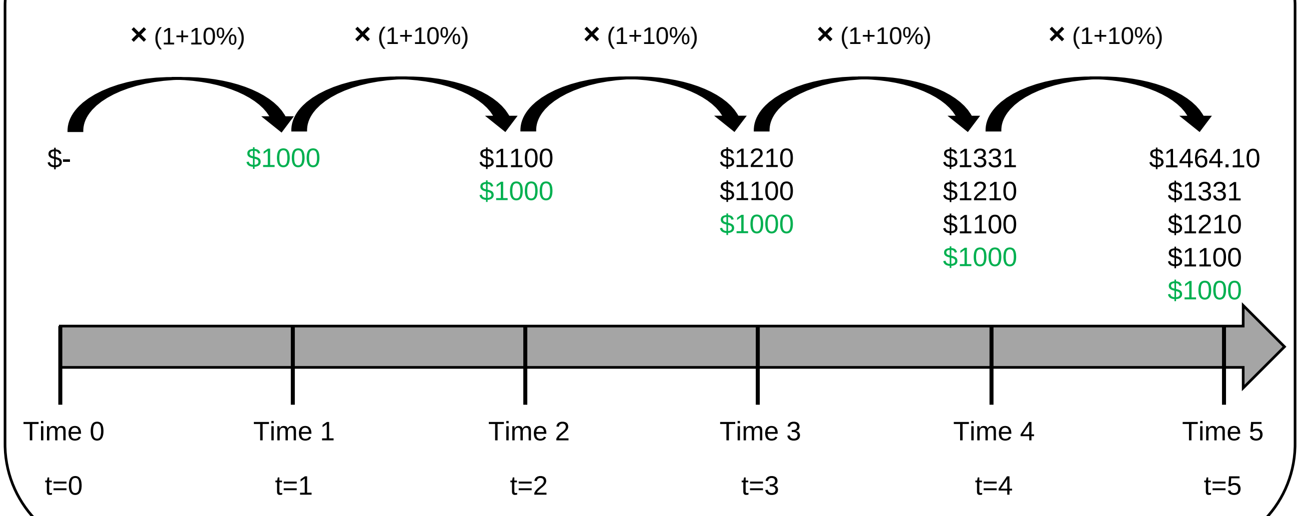
In practice, firms and individuals might receive, invest, or pay a *series* of cash flows rather than just receive, invest, or pay one **lump sum**. We now consider the future and present values of multiple cash flows over a period of time, but note that the *intuition remains the same*: we will continue to use the future value and present value formulas.

FUTURE VALUE OF MULTIPLE CASH FLOWS

EXAMPLE: You open a bank account that earns 10% annually. Beginning at the end of the year and then on the last day of each of the next 5 years, you deposit $1,000 into this account. How much will you have at the end of 5 years?

Here, it is important to visualize with a timeline. In green is the new deposit to the account made at the end of each period.

Figure 1: Future Value Multiple Cash Flows



At the end of 5 years, there is $1464.10 + 1331 + 1210 + 1100 + 1000 = **$6,105.10** in the account.

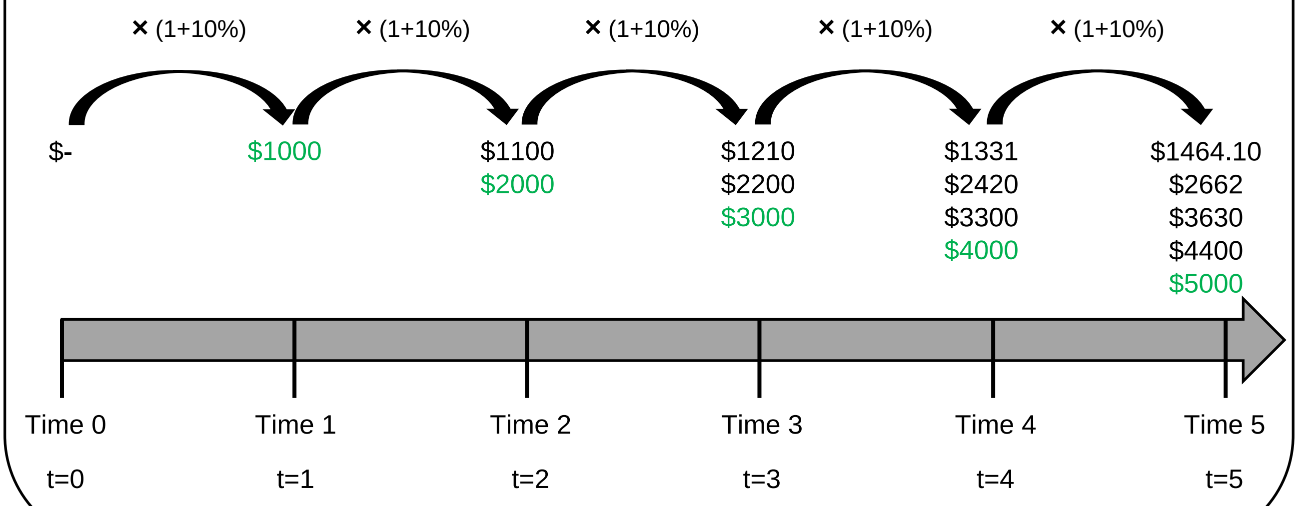
Using our FV formula, you could also write this as:

$$(1000 \times 1.1^4) + (1000 \times 1.1^3) + (1000 \times 1.1^2) + (1000 \times 1.1^1) + (1000 \times 1.1^0)$$

whereby each $1,000 deposit *compounds* its relevant number of periods. Note that because you waited a year to make your first payment, the initial $1,000 only compounds 4 times.

The amounts do not need to be constant. Assume now that each year you increase the deposited amount by $1,000 (i.e., deposit $1,000 then $2,000 then $3,000...). The timeline and formula would become:

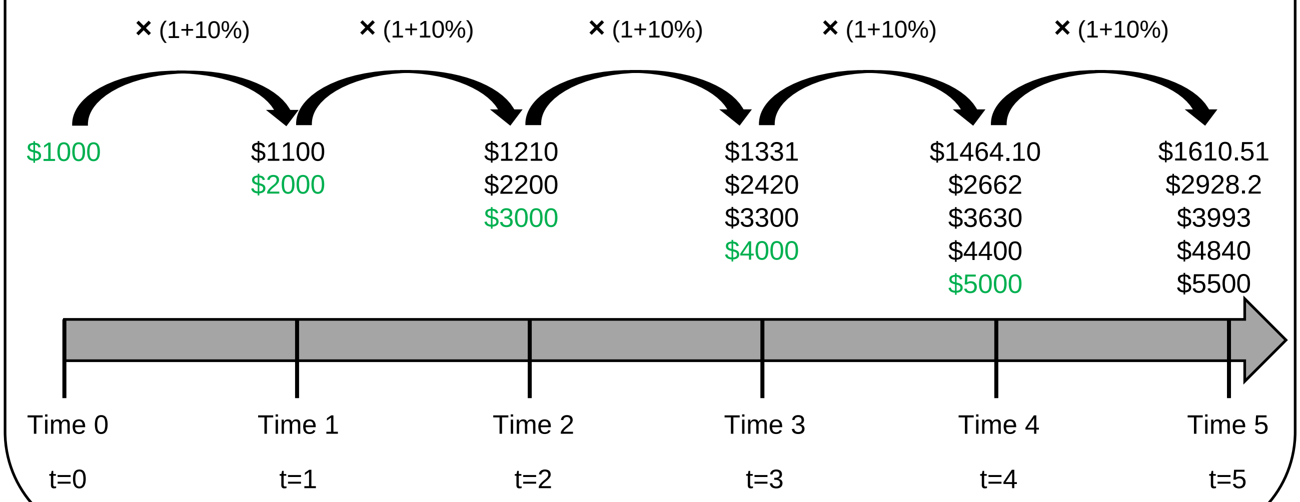
Figure 2: Future Value Multiple Growing Cash Flows
(end of period)



$$(1000 \times 1.1^4) + (2000 \times 1.1^3) + (3000 \times 1.1^2) + (4000 \times 1.1^1) + (5000 \times 1.1^0)$$

The future value in this case is **$17,156.10**. Note that the timing of the cash flow (at the beginning or ending of the period) matters. If you instead deposit $1,000 dollars *today*, then make your deposits at the *beginning* of each year:

Figure 3: Future Value Multiple Growing Cash Flows (beginning of period)



In our formula, notice the exponents are stepped up as each deposit compounds for an additional period:

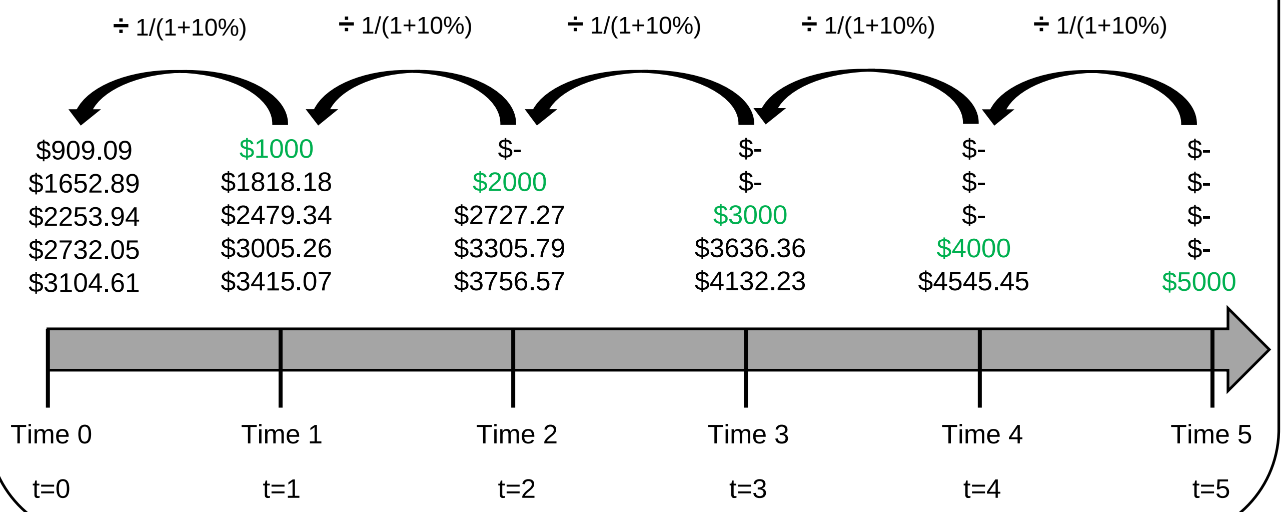
$$(1000 \times 1.1^5) + (2000 \times 1.1^4) + (3000 \times 1.1^3) + (4000 \times 1.1^2) + (5000 \times 1.1^1)$$

The future value here is **$18,871.71**. Despite contributing the same $15,000 to the account, you have $18,871.71 – $17,156.10 = $1,715.61 more in this case because of the additional year of compounding.

Present Value of Multiple Cash Flows

Similarly, we can use the present value formula to find the present value of multiple cash flows, or what a series of cash payments to be received in the future are worth *today*. This is the foundation of stock, bond, and company valuation, since each "generate" multiple cash flows through time. We call this process and similar applications **discounting cash flows**.

Figure 4: Present Value Multiple Growing Cash Flows
(end of period)



Therefore, the series of deposits made at the end of each year ($1000, $2000, $3000, $4000, and $5000) over the next five years is worth

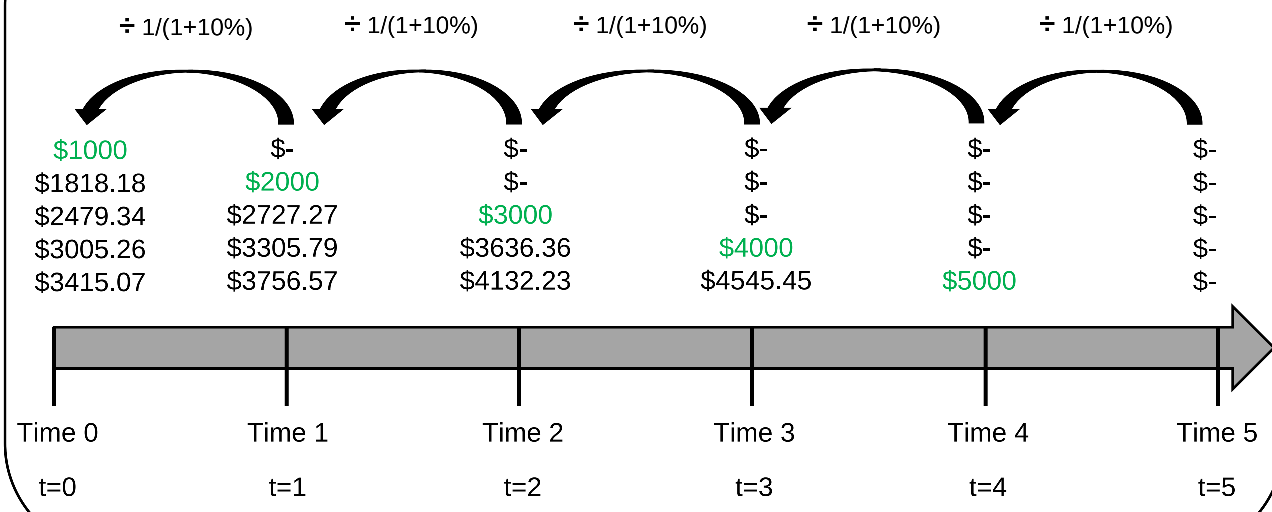
$909.09 + 1652.89 + 2253.94 + 2732.05 + 3104.61 = **$10,652.58**

today assuming a discount rate of 10%. By the present value formula:

$$\frac{1000}{(1+0.10)^1} + \frac{2000}{(1+0.10)^2} + \frac{3000}{(1+0.10)^3} + \frac{4000}{(1+0.10)^4} + \frac{5000}{(1+0.10)^5}$$

which gives the same answer. If the deposits are made at the *beginning* of the year:

Figure 5: Present Value Multiple Growing Cash Flows
(beginning of period)



This results in a present value of **$11,717.85**. This is greater than in the end-of-period case (PV = **$10,652.58**), given the deposits are closer to the present (and a dollar today is worth more than a dollar in the future).

APPLICATIONS OF MULTIPLE CASH FLOWS

PRACTICE: The Colorado Company won a lawsuit that entitles them to collect $100,000 a year for the next 30 years, beginning one year from now. This is known as a **structured settlement**. Assuming the firm can invest the money they receive at 9% per year:

(1) How much will they have at the end of 30 years?
(2) How much will they have if they receive their first payment *today* with future payments one year from today and every year thereafter?

A financial services company approaches The Colorado Company and offers to "buy" this structured settlement from them. What is the minimum amount that The Colorado Company should accept under both the beginning- and end-of-period scenarios? *Why* sell these cash flows? Why would a firm offer to buy these cash flows?

SOLUTION: First, we recognize that this is a future value question with many periods, and draw a timeline to help visualize the timing and frequency of payments:

And the formula is:

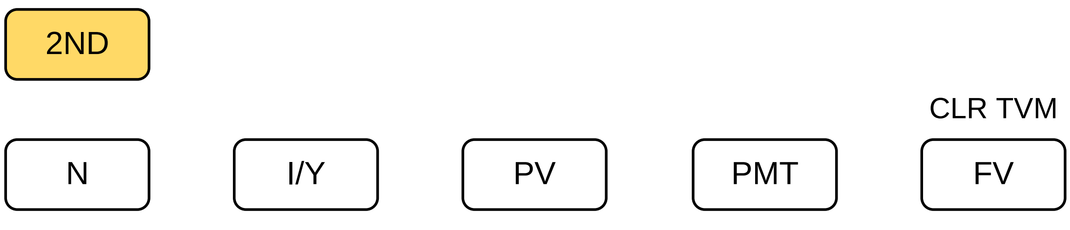
$$(100k \times \qquad\qquad) + (100k \times \qquad\qquad) + \cdots + (100k \times \qquad\qquad) + (100k \times \qquad\qquad)$$

As we did above, we can compute the future value by adding up all 30 of these individual sums. Yet, 30 periods is a lot to compute by hand. Fortunately, we have a formula that we can use for *fixed payments* that remain the same each period:

$$FV = PMT\left[\frac{(1+r)^{t}-1}{r}\right]$$

$$FV = \qquad\qquad \left[\frac{\left(1+\qquad\qquad\right)\quad -1}{\qquad}\right] =$$

The calculator can also compute this value, by using the **PMT** key:



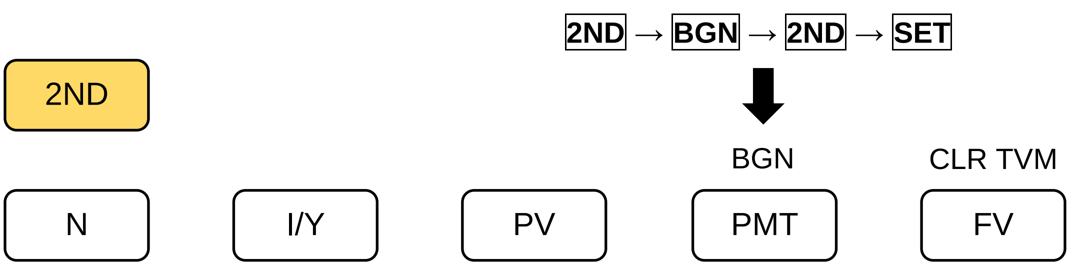
If the payments are at the beginning of the period, the timeline and formula become:

$$(100k \times \qquad\qquad) + (100k \times \qquad\qquad) + \cdots + (100k \times \qquad\qquad) + (100k \times \qquad\qquad)$$

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right] \times (1+r)$$

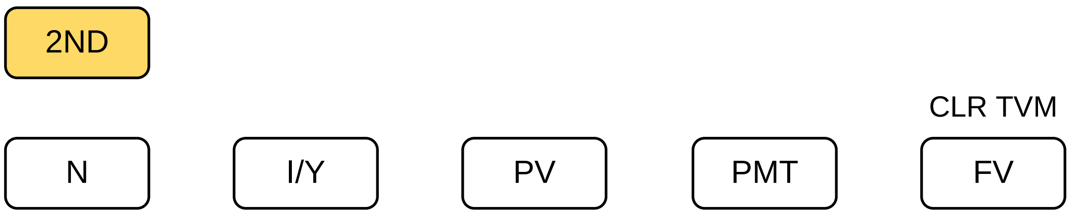
$$FV = \qquad\qquad \left[\frac{(1+ \qquad\qquad)^{\quad} - 1}{\qquad}\right] \times (1+ \qquad) =$$

The calculator inputs are:

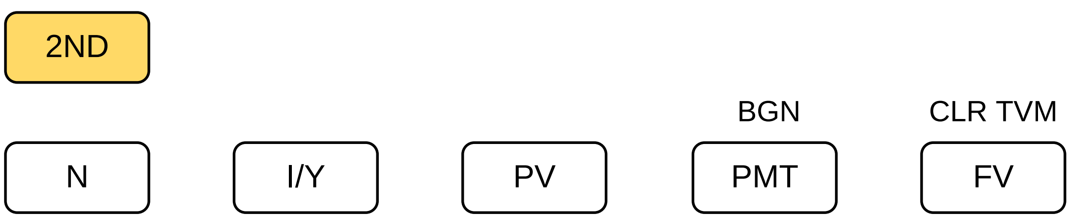


> ⚠ Be careful to check that your calculator is in the correct mode, only showing BGN when payments are at the *beginning* of the period. To toggle:
>
> **2ND** → **PMT** key→ **2ND** → **ENTER** key

This gives the same answer as the mathematical expression above. To determine what the end-of-period series of cash flows is worth in order for Colorado to sell them to the financial services firm, we compute the present value with the inputs:



If payments are received at the beginning of the period,



INTERPRETATION: To summarize:

FV end-of-period payments	
FV beginning-of-period payments	
PV end-of-period payments	
PV beginning-of-period payments	

The firm should only sell their cash flows if they are offered at least ______________ if payments are made at the end of the year, or ________________ if payments are at the beginning of the year.

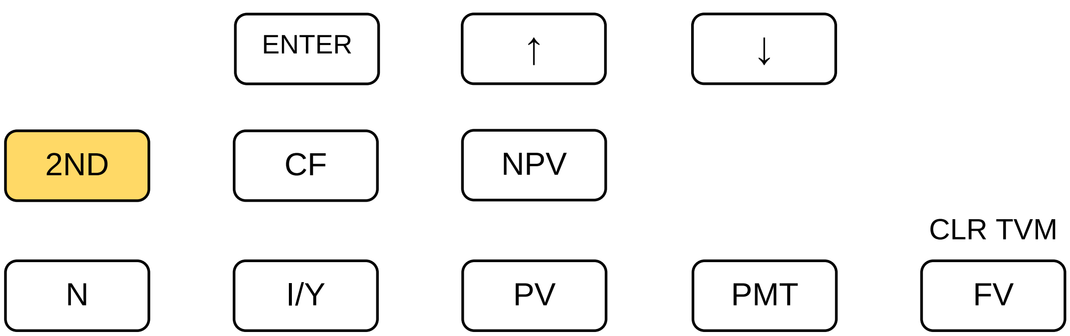
PRACTICE: An investor makes the following annual investments in the stock market:

Year 1	$2,000
Year 2	$3,000
Year 3	$4,000
Year 4	$4,000

What are these cash flows worth today? Why can't we just add up the annual values to determine what they are worth today? How much will the investor have in this account after 4 years? To address these questions, assume first investments are made (1) at the end of the years and then (2) at the beginning of the years. Assume the stock market returns 7.5% per year.

SOLUTION: Here, we have **uneven cash flows**. Because the payments vary, we cannot use the **PMT** key on our calculator, but we can use the built-in cash flow worksheet:

Part 1: Assuming end-of-period cash flows



Keystrokes	**Explanation**
CF	The "cash flow" key, ***CFo=*** appears on the screen
2ND CLR WORK	"Clear worksheet" containing all cash flows (above **CE|C** key)
CFo = 0 Enter ↓	Cash flow at time 0: zero, because payments are at the end
C01 = -2000 Enter ↓	Cash flow at time 1: -$2000, a cash outflow
F01 = 1 Enter ↓	Frequency of the first cash flow: once
C02 = -3000 Enter ↓	Cash flow at time 2: -$3000
F02 = 1 Enter ↓	Frequency of the second cash flow: once
C03 = -4000 Enter ↓	Cash flow at time 3: -$4000
F03 = 2 Enter	Frequency of the third cash flow: twice

NPV	"Net Present Value" button
I = 7.5 Enter ↓	The interest rate
CPT NPV	Compute the "Net *Present* Value"

The present value is $ (end-of-period)

To find the FV of these cash flows, CE|C 2ND CLR TVM then enter the following in the TVM keys: PV = , N = , I/Y = CPT FV.

The future value is $ (end-of-period)

Part 2: Assuming beginning-of-period cash flows

Keystrokes	Explanation
CF	The "cash flow" key, ***CFo=*** appears on the screen
2ND CLR WORK	"Clear worksheet" containing all cash flows (above CE|C key)
CF_0 = -2000 Enter ↓	Cash (out)flow at time 0: -$2000, payments are at the beginning
C01 = -3000 Enter ↓	Cash flow at time 1: -$3000
F01 = 1 Enter ↓	Frequency of the first cash flow: once
C02 = -4000 Enter ↓	Cash flow at time 2: -$4000
F02 = 2 Enter ↓	Frequency of the second cash flow: twice
NPV	"Net Present Value" button
I = 7.5 Enter ↓	The interest rate
CPT NPV	Compute the "Net *Present* Value"

The present value is $ (beginning-of-period)

To find the FV of these cash flows, CE|C 2ND CLR TVM and 2ND BEG 2ND SET CE|C to make sure the payments are set to BGN then enter the following in the TVM keys: PV = , N = , I/Y = CPT FV.

The future value is $ (beginning-of-period)

INTERPRETATION: To summarize, we note that present values and future values are both greater with beginning-of-period cash flows than end-of-period cash flows:

FV end-of-period payments	
FV beginning-of-period payments	
PV end-of-period payments	
PV beginning-of-period payments	

ANNUITIES & PERPETUITIES

ORDINARY ANNUITIES AND ANNUITIES DUE

Annuities are streams of cash flows that are fixed amounts (i.e., $100 every year) or grow at a fixed rate (i.e., 2% per year: $100, then $102, then $104.04) over a period of time. When the payments are at the end of the period, it is called an **ordinary annuity**. If the payments are at the beginning of the period, it is called an **annuity due**.

We've seen how to calculate the PV of "fixed amount" annuities with the **PMT** key above. But what if the payments are growing at a "fixed rate"? For an **ordinary annuity**, we can use the formula:

$$Growing\ Annuity\ PV = C \times \left[\frac{1 - \left(\frac{1+g}{1+r}\right)^t}{r - g}\right]$$

where

C = the first payment received one period from now

r = the discount rate

g = the fixed growth rate of the payments

t = the number of periods

To convert the results of the *Growing Annuity PV* formula into an **annuity due**, we simply apply the following formula:

$$Annuity\ Due\ PV = Ordinary\ Annuity\ Value \times (1 + r)$$

PRACTICE: You've won the lottery! You will be paid annually for the next 20 years. The first payment to you is $200,000, which you will receive one year from now. Every year thereafter, your payment will grow by 5% per year. What is the present value to you if you discount these cash flows by 11%? What if the first payment is received *today*, with all other payments received in one year intervals?

SOLUTION: First, we determine the present value of the **ordinary annuity**. By the formula

$$Growing\ Annuity\ PV = C \times \left[\frac{1 - \left(\frac{1+g}{1+r}\right)^t}{r - g}\right]$$

We have:

$$Growing\ Annuity\ PV = \qquad\qquad \times \left[\frac{1 - \left(\frac{1 + \qquad\qquad}{1 + \qquad\qquad}\right)}{\qquad - \qquad}\right]$$

$$Growing\ Annuity\ PV =$$

If payments are received beginning today, we know the present value of this sum is:

$$Annuity\ Due\ PV = Ordinary\ Annuity\ Value \times (1 + r)$$

$$Annuity\ Due\ PV = \qquad\qquad\qquad\qquad \times (1+ \qquad\qquad)$$

$$Annuity\ Due\ PV =$$

INTERPRETATION: If we add up all the payments, beginning with \$200,000 then \$210,000 and so on as they grow at 5% per year, we would have a total of about \$6.6 million! Yet, our present values are much less than that. Remember, the guiding principle of TVM is that a dollar today is worth more than a dollar in the future. Even though our 20th payment may have grown to over \$505,000, it is 20 years away and equivalent to a much lower *present value* today.

From the *Oxford English Dictionary*, a definiton of discount is "*to disregard or rule out as unreliabile, false or irrelevant. Also: to give little credence to.*"

This is what we are doing when determining the present value of future cash flows. Given uncertainty, we "discount" whether future cash flows will actually be earned, and we "penalize" these cash flows for occuring in the future given we can't earn interest on them in the meantime.

Given this understanding of a discount rate as a measure of uncertainty, do we think "riskier" cash flows have higher or lower discount rates than less risky cash flows?

PERPETUITIES

A **perpetuity** is an annuity where the cash flows continue forever, or *perpetually*. To obtain the present value of a series of infinite and constant cash flows, we follow the formula:

$$PV_{perp} = \frac{C}{r}$$

PRACTICE: Preferred stock pays a constant dividend to the stockholder every period. How much would you be willing to pay for a preferred share if it pays \$4 annually, and you think this firm should offer a return of 10% given its level of risk?

$$PV_{perp} = \frac{C}{r} = \frac{\qquad}{\qquad} =$$

INTERPRETATION: We might choose that 10% "required return" because other similar risky firms offer that rate. If the firm is riskier, we might require a higher return. If less risky, we would be content to accept a lower rate of return.

If the cash flows associated with a perpetuity grow at a constant rate g, we use:

$$PV_{growing\ perp} = \frac{C}{r-g}$$

PRACTICE: A firm announces that it plans to pay a $0.25 dividend at the end of the year, and that it plans to grow that dividend by 3% each year forever. How much should you pay for this stock if you require a return of 5% on this company?

$$PV_{perp} = \frac{C}{r-g} = \frac{\qquad\qquad}{} =$$

What are some other things in finance that resemble an annuity? A perpetuity?

SUMMARY

Here, we introduced the concept of "multiple cash flow" and:

- Computed present and future values of multiple cash flows over many periods
- Discussed perpetuities and annuities, and the difference between the two
- Examined the effects of end-of-period (ordinary) and beginning of period (due) payments.

We will use discounted cash flow valuation to determine the value of stocks, bonds, and projects.

Additional examples are available in the Excel file: *Discounted Cash Flow* at www.josephfarizo.com/fin360.html.