University of Richmond
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Risk, Return, the Security Market Line, and Cost of Capital

FIN 360: Principles of Financial Management

RETURNS AND VARIANCES

An investor's return on an investment is their total gain or loss from holding the financial asset. It is the sum of a security's:

1. **Income component** – the dividends and/or interest received over the holding period.
2. **Capital gain** (or **capital loss**) **component** – the change in value of the investment over the holding period.

For stocks, we can determine total returns by adding up the **dividend yield** and **capital gains yield**, which represent the income and capital gain components, respectively:

$$Total\ \%\ Return = Dividend\ Yield + Capital\ Gain\ Yield$$

$$Total\ \%\ Return = \frac{D_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

EXAMPLE: You acquire stock for \$56.65 at the beginning of the year, and sell for \$51.24 at the end of the year. It paid a total of \$1.60 in dividends, which we can assume occured at the end of the year. Its dividend yield, capital gains yield, and total return is:

$$Total\ \%\ Return = \frac{1.60}{56.65} + \frac{51.24 - 56.65}{56.65} = 2.824\% + -9.55\% = -6.726\%$$

This is a **realized return**, a return that *actually* occurred. **Expected returns** are what we believe a security will yield in the future. We estimate expected returns by considering what a security traditionally returned in various states of the world, and by estimating the probability a similar state of the world will occur in the future.

However, expected returns are only one part of the story. Securities, and **portfolios** of securities, are also characterized by their *risk*.

EXAMPLE: Let's find the expected return of a portfolio of stocks. Then, we'll characterize the risk of this portfolio by finding the **variance** and **standard deviation** of the returns. The variance and standard deviation give us an idea of the "spread" or "range" of possible returns. The wider this spread, the greater the risk. Consider the portfolio below:

State of Economy	Probability of State Occurring	Stock Returns		
		A	B	C
Boom	40%	10%	15%	20%
Bust	60%	8%	4%	0%
	100%			

We can find the expected return for a security by summing the products of the security's return in each state *r(s)* and the probability of that state *p(s)* occurring:

$$E(r) = \sum_{s=1}^{S} p(s)r(s) = p(s_1)r(s_1) + p(s_2)r(s_2) \cdots + p(s_S)r(s_S)$$

Or,

$$E(R_A) = \sum p(s)_A r(s)_A = (0.40 \times 0.10) + (0.60 \times 0.08) = 8.8\%$$

$$E(R_B) =$$

$$E(R_C) =$$

If we want to determine the expected return of a portfolio $E(R_P)$, given we put an equal 1/3 of our money in each stock:

$$E(R_P) = \frac{1}{3}(8.8\%) + \frac{1}{3}(\qquad\qquad) + \frac{1}{3}(\qquad\qquad) = 8.392\%$$

This characterizes the expected return of the portfolio. But the risk is also crucially important. To "quantify" our level of risk, we compute the standard deviation of the portfolio by taking the square root of the variance. To find the variance:

(1) Find the return of the portfolio in each state of the economy.
(2) Find the difference between how the portfolio performs in each state of the economy and the portfolio expected return you just computed.
(3) Square the differences from part (2)
(4) Multiply the squared differences from (3) by the probability of their state occurring.
(5) Sum the values you find in part (4)

This gives you the variance. These same steps are summarized in the formula:

$$Var(r) = \sigma^2 = \sum_{s=1}^{S} p(s)[r(s) - E(r)]^2$$

Take the square root of this value to find the standard deviation.

$$SD(r) = \sigma = \sqrt{Var(r)}$$

Assuming we have 1/3 of our portfolio in each stock:

$$Return_{Boom} = \frac{1}{3}(10\%) + \frac{1}{3}(\qquad\qquad) + \frac{1}{3}(\qquad\qquad) = 14.99\%$$

$$Return_{Bust} = \frac{1}{3}(8\%) + \frac{1}{3}(\qquad\qquad) + \frac{1}{3}(\qquad\qquad) = 4.00\%$$

I. State of Economy	II. Probability of State Occurring	III. Stock Returns A	IV. Stock Returns B	V. Stock Returns C	VI. Return of Portfolio in Each State
Boom	40%	10%	15%	20%	
Bust	60%	8%	4%	0%	

Given we already calculated the expected return of the portfolio $E(R_P)$, we find the difference between the return in each state and the expected return. We then square that difference and sum to obtain the variance. Finally, we take the square root for the standard deviation.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
State of Economy	Probability of State Occurring	Stock Returns			Return of Portfolio in Each State	Squared Difference	Squared Difference × Prob.
		A	B	C			
Boom	40%	10%	15%	20%			
Bust	60%	8%	4%	0%			
						VAR =	
						SD =	

⚠ Some of these numbers will be very small. Store them in your calculator rather than rounding intermediate steps. You should show at least 6 decimal places for variance. Variance is usually shown as a decimal. Standard Deviation is usually converted to a percent.

INTERPRETATION: The **standard deviation** is an important measure because it tells us how widely dispersed our returns are expected to be. We usually use it *with* the expected return. Our interpretations is as follows:

- There is about a 99% chance our portfolio will return within ± 3 standard deviations of the expected return.

- There is about a 96% chance our portfolio will return within ± 2 standard deviations of the expected return.

- There is about a 68% chance our portfolio will return within ± 1 standard deviations of the expected return.

Thus, the bigger our standard deviation, the greater likelihood our portfolio can produce returns outside of what we expect, and the riskier the portfolio is. Anything above or below 3 standard deviations from the expected return by this methodology would be unusual.

For additional practice and an expected return and standard deviation calculator, see the Excel file *Expected Return and Risk* at www.josephfarizo.com/fin360.html.

RISK, DIVERSIFICATION, AND PORTFOLIO RISK

Risk is uncertainty, or *dispersion* around our expectations. This could either be in the positive or negative direction. Risk is classified into two broad categories: **systematic risk** and **unsystematic risk**.

1. **Systematic risk** affects many assets – surprises that affect the market overall. This includes unemployment, interest rates, inflation, geopolitical risks, and energy prices.

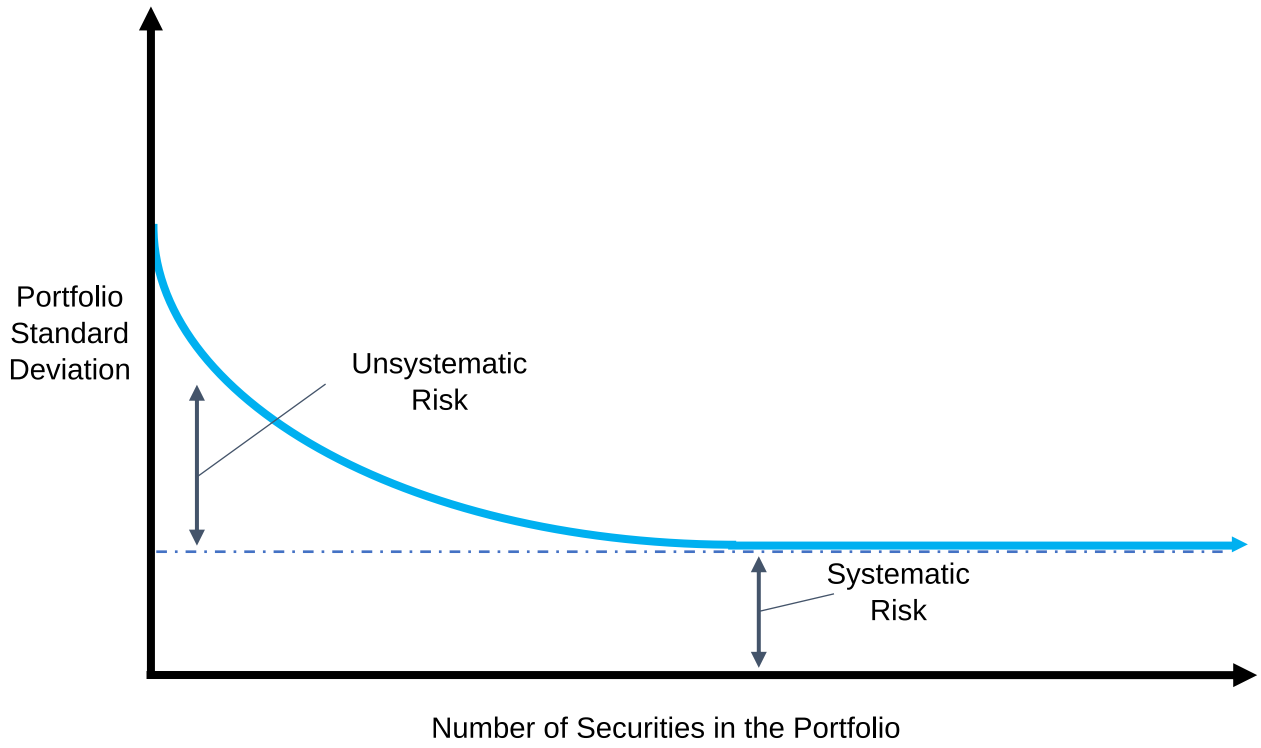
2. **Unsystematic risk** is unique to an individual firm. This includes the financial risks (possibility of default), managerial risks (fraud), labor-related (strikes), and other firm-specific uncertainties.

Diversification is the process of developing a portfolio or collection of assets such that you reduce your risk. Crucial to diversification is one important understanding:

Diversification can practically eliminate **unsystematic risk**. Diversification cannot eliminate **systematic risk**.

In other words, if you hold a portfolio of 500 stocks (with the same dollar amount invested in each stock), you are *still* exposed to the strength of the economy, interest rates, inflation, and other market-wide issues. However, if you hold a portfolio of 500 stocks, any one stock's own risk matters very little to you overall, and one stock's (or stocks') losses can be offset by another stock's (or stocks') gains.

Figure 1: Portfolio Diversification and Risk



THE SECURITY MARKET LINE AND BETA

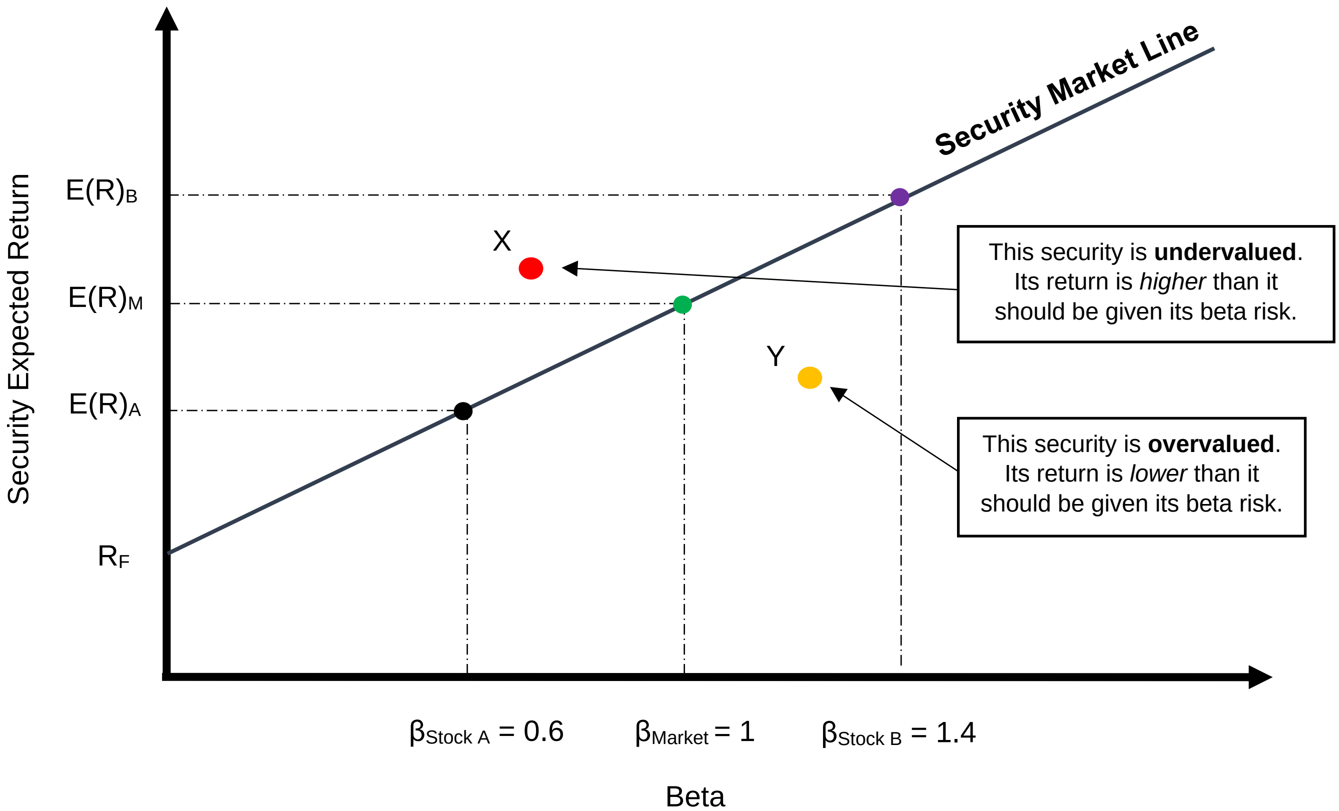
Beta is a measure of **systematic** risk. As we've seen before, it tells us how a stock moves relative to the market overall. Recall the **Capital Asset Pricing Model** (**CAPM**) tells us that the expected rate of return (and therefore the rate of return required on an investment) is defined as:

$$E(R) = r_f + \beta(E(R_M) - r_f)$$

We can depict the CAPM relationship on what is known as the **Security Market Line** (**SML**) – a line connecting the expected return of a security to the expected return of the market.

It shows a *linear relationship* between a security's expected or required rate of return and the security's beta.

Figure 2: The Security Market Line



Notice the following features of the SML:

- The line begins at the risk-free rate of return and is drawn through the **market portfolio** that consists of all assets. It has a beta = 1.

- Stock A has a lower expected return than B because it has a lower beta.

- Stock X is undervalued because it has a return greater than that predicted by the CAPM. Investors would buy up this stock (paying more and more for it, reducing its return) such that it falls on the SML.

- Stock Y is overvalued because it has a return lower than predicted by the CAPM. Investors would sell out of this stock (reducing its price and increasing its return to new investors) such that it falls on the SML.

Given an understanding of this relationship between return and systematic risk, we can consider next the appropriate required rate of return (and therefore *cost*) of capital.

> ⚠ The CAPM and SML are controversial. Yet, they are widely used in practice as we will see in the next section. There is much academic evidence that questions its validity and shows that securities don't always follow the rules that theory says they should.

COST OF CAPITAL

We have now discussed the required and expected returns on both equity and debt that a firm issues. We know we can use the **CAPM** to determine the rate of return required by investors on a firm's equity. Similarly, we might use a firm's bonds' **yield to maturity** to determine the rate of return required by investors in the firm's debt (lenders).

The rates of return the providers of capital (that is, the *investors* in the firm's securities) is therefore a *cost* to the firm – and the firm will be expected to undertake actions such that it can achieve and deliver on these required rates of return.

- **Cost of Equity** is the return that equity investors require on their investment in the firm (determined by the **CAPM**)

- **Cost of Debt** is the return that bond investors – the lenders – require on the firm's debt (determined by the firm's bonds' **YTM**s)

WEIGHTED AVERAGE COST OF CAPITAL

A firm's **weighted average cost of capital** is the weighted average of the cost of equity and *after-tax* cost of debt. It is the overall return the firm must earn on its existing assets to maintain the firm's value, taking into account the firm's **capital structure**, or the percentage of the firm financed with equity and the percentage of the firm financed with debt.

A firm's market value can be represented as:

$$V = E + D$$

Where E is the market value of the firm's equity and D is the market value of the firm's debt. This implies that the percentage of the firm that is comprised of equity and the percentage of the firm that is comprised of debt is, respectively:

$$\frac{E}{V} = \frac{Shares\ Outstanding \times Price\ Per\ Share}{V} \quad \text{and} \quad \frac{D}{V} = \frac{Market\ Value\ of\ Debt}{V}$$

E/V and D/V are **capital structure weights**. Using these weights, we can determine that the Weighted Average Cost of Capital must be:

$$WACC = \frac{E}{V}R_E + \frac{D}{V}(1 - T_C)R_D$$

where R_E is the cost of equity by the CAPM, R_D is the cost of debt by the firm's bonds' YTM, and T_C is the corporate tax rate, given the tax advantage of debt (interest is paid *before* taxes are paid, but dividends are paid *after* tax and the firm receives no tax benefit for paying dividends).

EXAMPLE: Lafitte Inc. has 1.4 million shares outstanding, and its stock sells for $20 per share. The firm's debt has a total face value of $5 million, was issued 22 years ago but has 8 years to maturity. This debt is paying 12% coupons (annual rate, but paid semiannually) and its current price is 104.82% of par. The rate on T-Bills is 8%, the expected return of the market is 15%, and the beta is 0.74. Assuming a corporate tax rate of 21%, what is the firm's WACC?

SOLUTION: We begin with the formula for WACC, and compute each of its component parts:

$$WACC = \frac{E}{V}R_E + \frac{D}{V}(1 - T_C)R_D$$

E is the market value of the stock, which we know as the firm's **market capitalization**:

$$E = Shares\ Outstanding \times Price\ Per\ Share =$$

D is the market value of the debt. According to the problem, the bonds are trading at 104.82% of par:

$$D = Par\ Value \times Percentage\ of\ Par =$$

Given we have E and D, we know that V is the sum of the two, and E/V and D/V are the percentages of the firm comprised of equity and debt, respectively:

$$V = E + D$$

$$\frac{E}{V} =$$

$$\frac{D}{V} =$$

R_E is obtained using the CAPM formula:

$$R_E = r_f + \beta\big(E(R_M) - r_f\big) =$$

R_D is obtained by computing the YTM on the debt. From the problem, we know the following inputs:

N =

PV =

PMT =

FV =

<**CPT**> **I/Y** = $\times$ **2** = $= R_D$

Given the tax rate was provided by the problem, we can put it all together and arrive at:

$$WACC = \frac{E}{V} R_E + \frac{D}{V}(1 - T_C) R_D =$$

INTERPRETATION: We use the WACC as the appropriate discount rate for the firm's projects, assuming the risk of the project is similar to the risk of the firm. WACC reflects the risks of *both* primary sources of financing: debt and equity.

The cost of capital primarily depends on the *use* of the funds, not the source. It is inappropriate to use a firm's historical risk as a capital budgeting tool if the investment it is making has risks substantially different than the firm has historically been exposed to.

For example, if Firm A is considering acquiring Firm B, use Firm B's WACC as the appropriate discount rate for the incremental cash flows that Firm B generates in an NPV analysis.