University of Richmond
Robins School of Business™

Time Value of Money

Fin 360: Principles of Financial Management

INTRODUCTION TO TVM CONCEPTS

EXAMPLE: You open a bank account with $100 that pays 5% per year. How much do you have at the end of one year?

$$\$100 \times (1 + 0.05) = \$105 \qquad (1)$$

At the end of the year, you have $105 in that account. You decide to leave it at the bank for one more year. How much will you have at the end of the second year, assuming the bank again pays 5% per year in interest?

$$\$105 \times (1 + 0.05) = \$110.25 \qquad (2)$$

Substitute expression (1) into expression (2):

$$\$105 \times (1 + 0.05) = \$110.25$$
$$\$100 \times (1 + 0.05) \times (1 + 0.05) = \$110.25$$
$$\$100 \times (1 + 0.05) \times (1 + 0.05) = \$110.25$$
$$\$100 \times (1.05)^2 = \$110.25$$

The **present value (PV)** of $100 has grown to a **future value (FV)** of $110.25, given the **interest rate (r)** of 5% over two **periods (t)**. Put another way:

$$\mathrm{PV}(1 + \mathrm{r})^{\mathrm{t}} = \mathrm{FV}$$

We can also rearrange this formula with simple algebra and solve for the **PV** by dividing through by $(1+r)^t$, giving us:

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1 + \mathrm{r})^{\mathrm{t}}}$$

This is a useful formula if we know what the FV needs to be for some spending in the future, but are unsure of what the PV we should “invest” today is.

EXAMPLE: You plan on renting a new apartment in 3 years that will require a $2,000 deposit. The bank offers a safe investment that returns 6% per year. How much do you need to invest *today* to be able to withdraw $2,000 in 3 years?

First, identify the components:

FV = $2,000 **r = 6%** **t = 3** **PV = ?**

Next, plug the values into the formula, and solve:

$$PV = \frac{FV}{(1+r)^t}$$

$$PV = \frac{2000}{(1+0.06)^3}$$

$$PV = \frac{2000}{(1.06)^3} = \frac{2000}{1.191016} = \$1{,}679.24$$

The present value is $1,679.24, implying you need to invest this much *today* to have the $2,000 needed in the *future*.

Do you think you'd have to invest more or less money today if **r = 10%**? What can we say about the "present value" as interest rates get bigger?

THE TIME VALUE OF MONEY

These simple examples highlight a fundamental concept in finance known as the **time value of money (TVM)**.

The **time value of money (TVM)** is the idea that a sum of money *today* is worth more to you than that same sum of money in the *future* because of the ability of the money to earn a rate of return over time. Delaying your receipt of cash is a lost opportunity!

EXAMPLE: You win a raffle from the state lottery for $1,000. Your prize money can be collected anytime within one year. A local bank pays 5% interest per year. If you pick up the money *today*, you will have $1,000 + (5% of $1,000) = $1,050 in one year from today. If you wait one year, you'll only get the $1,000 with a whole year of missed interest.

A dollar today is worth more than a dollar in the future.

TVM is the single most important concept in this course, and a central theme in finance and *all* of business. It has numerous *real-world* applications:

1. **Stock valuation**: determine what a stock is worth based on the present value of its dividends or "free cash flows" the firm generates
2. **Bond valuation**: determine what a bond is worth based on the present value of its interest payments and the principal to be paid at maturity
3. **Company valuation**: determine what a firm is worth based on the present value of its future cash flows
4. **Project valuation**: determine whether a factory should be built or another company acquired given the cash flows it is projected to yield
5. **Personal finance**: determine how much is needed to save *today* for a down payment on a home or how much you need to save to retire

With the framework of the previous examples, as well as an understanding of just how important TVM is, let's dive deeper into examples and applications.

FUTURE VALUES AND COMPOUNDING

Recall from our examples above the formula:

$$\mathrm{FV} = \mathrm{PV}(1 + \mathrm{r})^{\mathrm{t}}$$

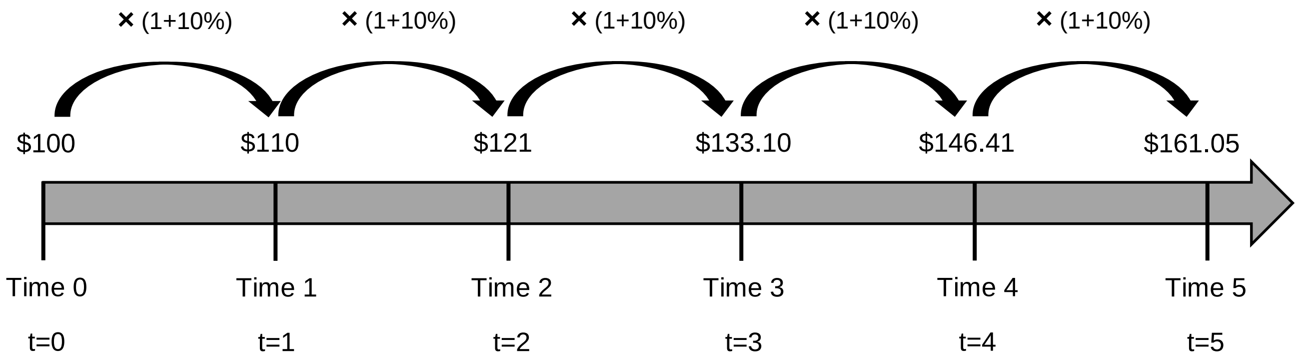
This formula illustrates a very important feature known as **compounding or compound interest**, which is the accumulation of money based on "*new* interest earned on interest *previously* earned." This compounding effect is evident in the $(1+r)^t$ portion of the equation (where $(1+r)^t$ is referred to as the **future value factor)**.

EXAMPLE: Suppose you invest $100 in an account which earns 10% per year, or interest *compounds annually*. After 5 years, we know we will have:

$$\mathrm{FV} = \mathrm{PV}(1 + \mathrm{r})^{\mathrm{t}} = 100(1 + 0.10)^5 = \$161.051$$

We would have *less* if we earned **simple interest**, or just 10% of the original principal each year. That would result in an ending value of just $150: 100 + [(10% of 100) × 5] = $150.

Figure 1: Future Value Timeline



Notice how compounding allows you to earn additional interest on top of both the **principal** amount invested (the original $100), and the **simple interest** earned in the previous period.

We can also write (as we've seen before):

$$FV = \$100 \times 1.10 \times 1.10 \times 1.10 \times 1.10 \times 1.10$$

$$FV = \$100 \times (1.10)^5$$

$$FV = \$161.05$$

Compounding is *powerful*, particularly as we increase the number of periods *t*. Consider the figure below which demonstrates the "interest earned on interest" for the previous example:

Figure 2: The Power of Compounding

I. Year	II. Beginning amount	III. Simple Interest (10% of principal)	IV. Compound Interest (10% on interest earned)	V. Total Interest Earned (III + IV)	VI. Ending Amount (II + V)
1	$100 (principal)	$10	$0	$10	$110
2	110	10	1	11	121
3	121	10	2.1	12.10	133.10
4	133.10	10	3.31	13.31	146.41
5	146.41	10	4.64	14.64	161.05
⋮	⋮	⋮	⋮	⋮	⋮
50	10,671.90	10	1057.19	1067.19	11,739.09
⋮	⋮	⋮	⋮	⋮	⋮
100	1,252,783	10	125,268.3	125,278.3	1,378,061

❓ How much would the 10% investment above grow to after 50 years of simple interest? With compound interest? How does this difference change as years pass?

Crucially, banks almost always pay compound interest. Investments in the stock and bond markets grow with compounding. Balances on debts like mortgages and credit cards also experience compounding, in the sense that carried balances from one period to the next don't go away!

Present Value and Discounting

By revisiting the present value formula:

$$PV = \frac{FV}{(1+r)^t}$$

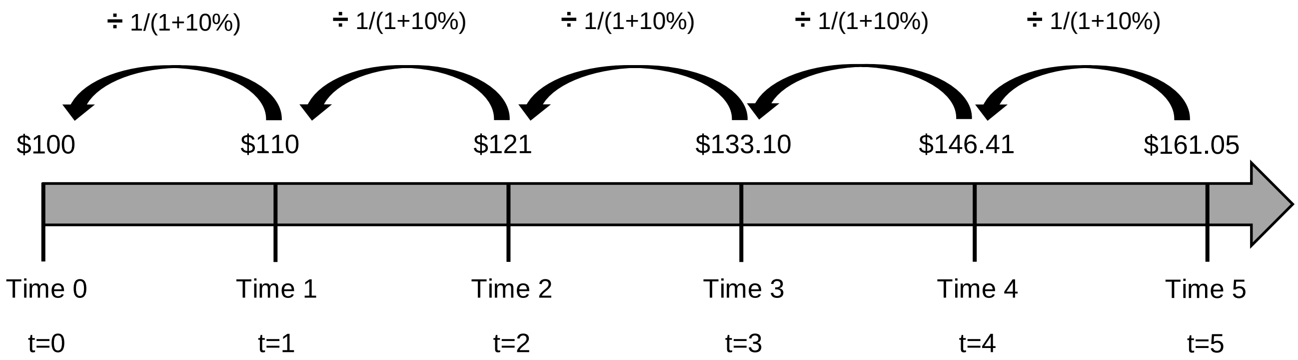
we can find what some value in the future is worth – *today*. In this context, r is referred to as the **discount rate**. Functionally, it is the same as the r in the future value formula, but the terminology changes. The $1/(1+r)^t$ portion of the equation is referred to as the **present value interest factor**.

Terminology for *r*		
Convert from PV to FV	Interest rate, growth rate	Growing, compound growth, compounding
Convert from FV to PV	Interest rate, discount rate	Discounting

EXAMPLE: Suppose we wish to have $161.05 in an account (that earns 10% per year) in 5 years. How much do we need to invest today?

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1+\mathrm{r})^{\mathrm{t}}} = \frac{\$161.05}{(1+0.10)^5}$$

Figure 3: Present Value Timeline



We can also write:

$$PV = \frac{\$161.05}{1.10 \times 1.10 \times 1.10 \times 1.10 \times 1.10}$$

$$PV = \frac{\$161.05}{(1.10)^5}$$

$$PV = \$100$$

APPLICATIONS OF TVM

In practice, using a financial calculator (or Excel and other software) is common in many cases.

USING A FINANCIAL CALCULATOR

PRACTICE: A firm estimates that 17 key employees will be retiring in 8 years, and each employee is due a cash retirement gift of $10,000. The firm can invest money today in an account earning 4% annually. How much do they need to put in the account today to be able to fund these bonuses?

SOLUTION: This is a PV problem, with the following information given.

FV = × =

r or **I/Y** =

t or **N (Number of periods)** =

PV = ?

Setting up the calculator:

1. Display the maximum number of decimal places:

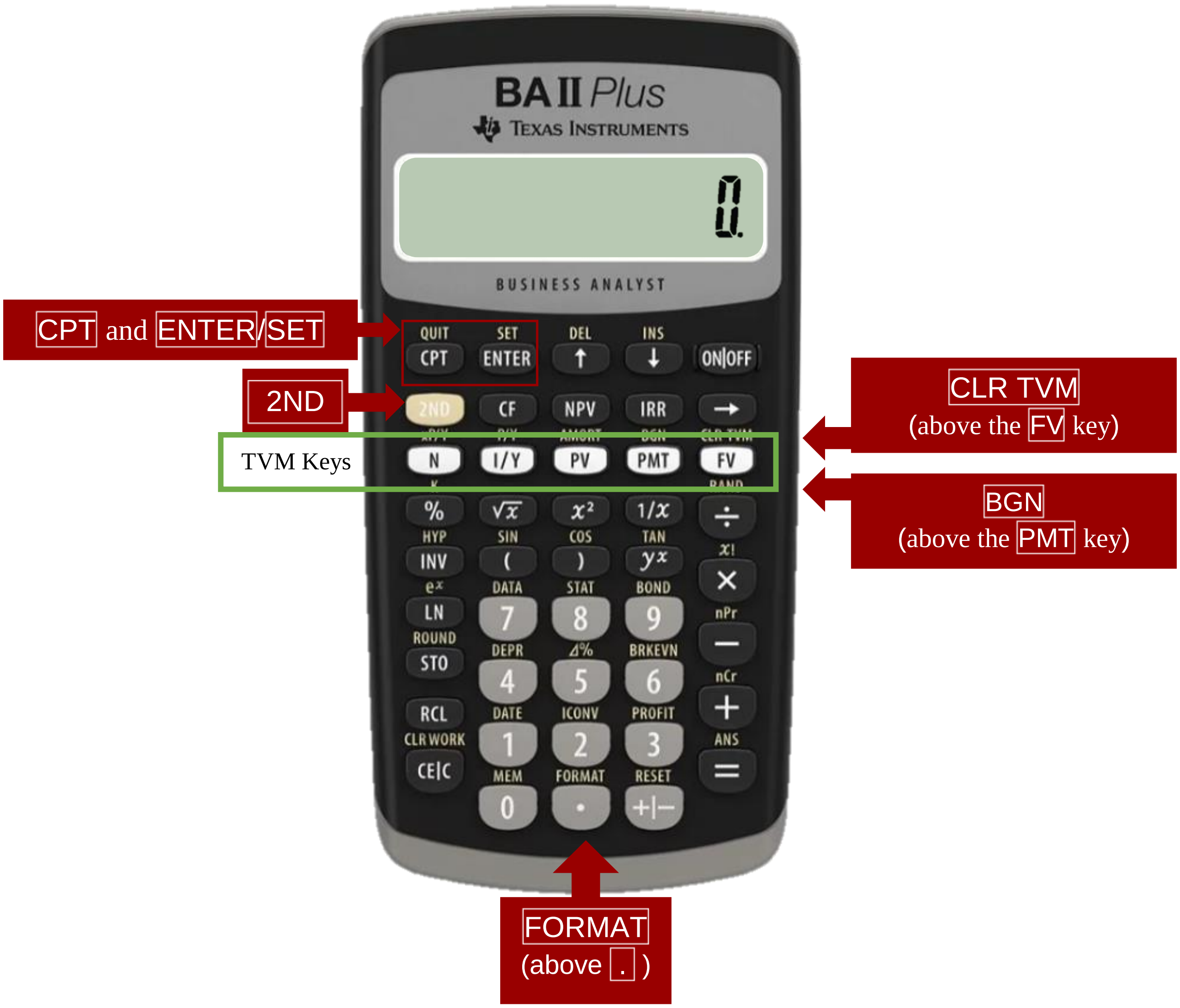
   2ND → **FORMAT** → use ↑ ↓ arrows to navigate to **DEC** → **9** → **ENTER**

2. Make sure there is no "BGN" above the zero on your screen.

   2ND → **BGN** → **2ND** → **SET**

⚠ Before *every* TVM problem, you *must* clear the stored values in the TVM keys:

2ND → **CLR TVM**



For this problem, we perform the following keystrokes (pressing the number *first* then the relevant key):

CPT

2ND

N	I/Y	PV	PMT	FV (CLR TVM)
8	4	<CPT>		17 × 1000 = 170,000
Number of years	*Interest rate*			*Future value needed*

Which matches the formula:

$$\text{PV} = \frac{\$10{,}000 \times 17}{(1 + 0.04)^8} = \$$$

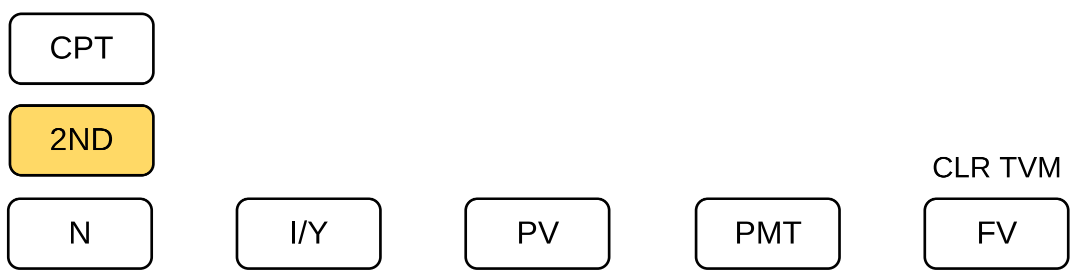
> The calculator gave us a *negative* present value. This is because the firm must invest (an outflow) that money to receive the future value (an inflow) in the future. Of course, the "true answer" is positive. The sign is just the calculator telling us the direction of the cash flow. We <u>do not</u> use that negative sign in the formula.

INTERPRETATION: The firm must set aside $124,217.33 in order to have the $170,000 needed in 8 years, assuming that money can grow at 4% annually.

ADDITIONAL PRESENT VALUE & FUTURE VALUE EXAMPLES

PRACTICE: A firm plans to pay $13 million in cash to begin the construction of a factory in 4 years. They have identified an investment opportunity that will yield them 8% annually in the meantime. (a) If the firm invests $9 million today, will they be able to afford the $13 million in 4 years? (b) Exactly how much would they need to invest today in order to have the $13 million when it is needed?

SOLUTION: This is a two part problem, highlighting both a present value and future value approach. In part (a), we know the firm has $9 million today and we wish to determine what that will be worth in 4 years.



By the formula, we get the same answer:

$$FV = PV(1 + r)^t = \qquad\qquad (\ 1\ +\ \qquad\ )^{4} \ =$$

Would the firm be able to afford the $13 million in 4 years?

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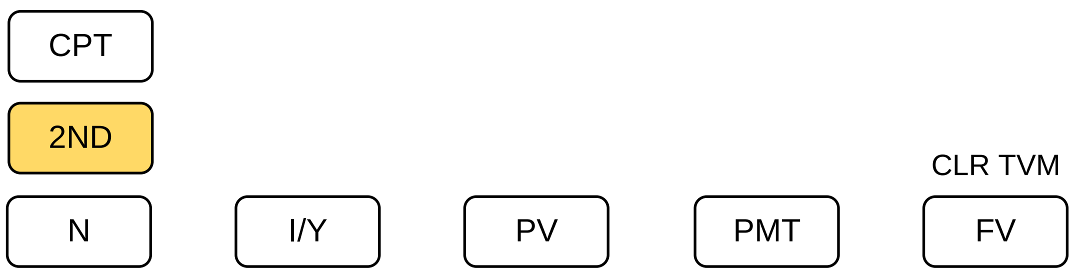
In part (b), we'll determine the exact amount the firm would need to invest today in order to have the $13M in 4 years. This is a present value problem: we know what they need in 4 years, we need to figure out how much they need to invest in the *present*.



By the formula, we get the same answer:

$$PV = \frac{FV}{(1 + r)^t} = \frac{\qquad\qquad}{(1 + \qquad\qquad)} =$$

Exactly how much will the firm need to invest today (at the 8% interest rate) to be able to afford the $13 million in 4 years?

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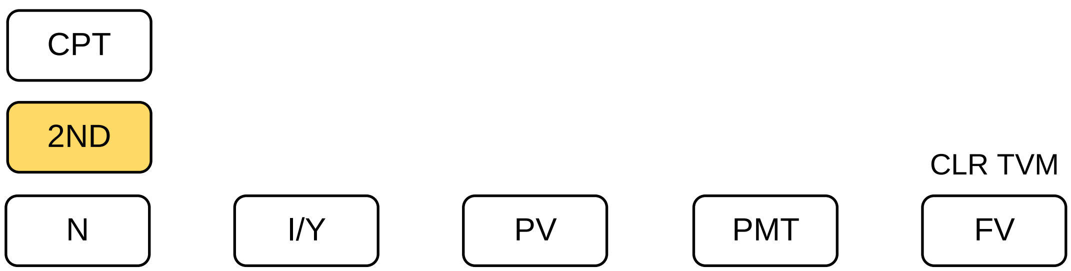
Present Values in the Future

Practice: How much do you need to invest in 3 years if you plan on having $30,000 in 10 years in order to purchase your dream motorcycle, the Harley-Davidson Street Glide, assuming your investment will be able to grow at 12% per year?

Solution:



Interpretation: Be mindful of the terms "present value" and "future value". The "present value" doesn't necessarily mean today. It is technically defined by the formula: a function of the future value, interest rate, and number of periods.

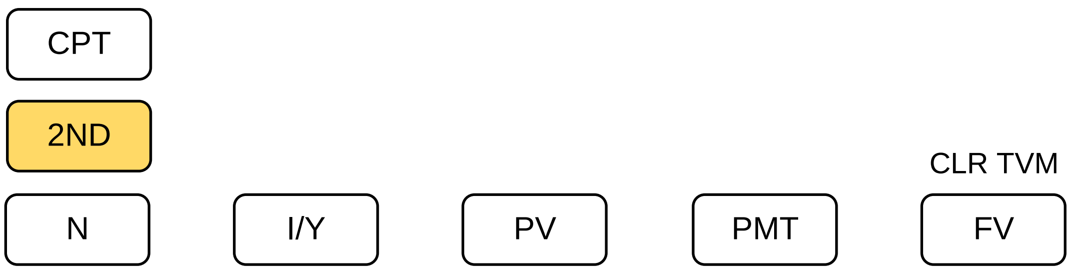
> ⚠ "Future (present) values" do not need to be in the "future (present)." These terms just tell us where we are *relative* to the other present or future values.

Solving for Other Inputs

The formulas for present value and future value imply that we might solve for the time periods N and the interest rate I/Y when we know the present value and future values.

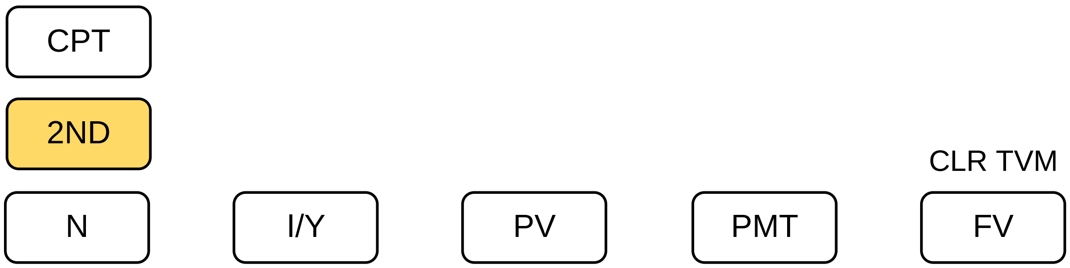
Practice: In 3 years you expect to begin a new career, and your employer will give you a signing bonus of 10% of your \$78,000 base salary. At that time, you plan on investing that bonus at 3.8917% per year until you have \$12,000, enough for you to take a 11-month backpacking trip through Europe. In how many years from now will you be able to afford this trip? What would the interest rate need to be if you needed \$13,000 instead of \$12,000?

Solution: Here, we are looking for the number of years from *now* where we will be able to afford this trip.



Is the computed "N" the answer to the question?

How much would the interest need to be if you instead needed $13,000 for the trip?



Additional TVM examples are available in the Excel file *TVM Practice Problems* at www.josephfarizo.com/fin360.html.